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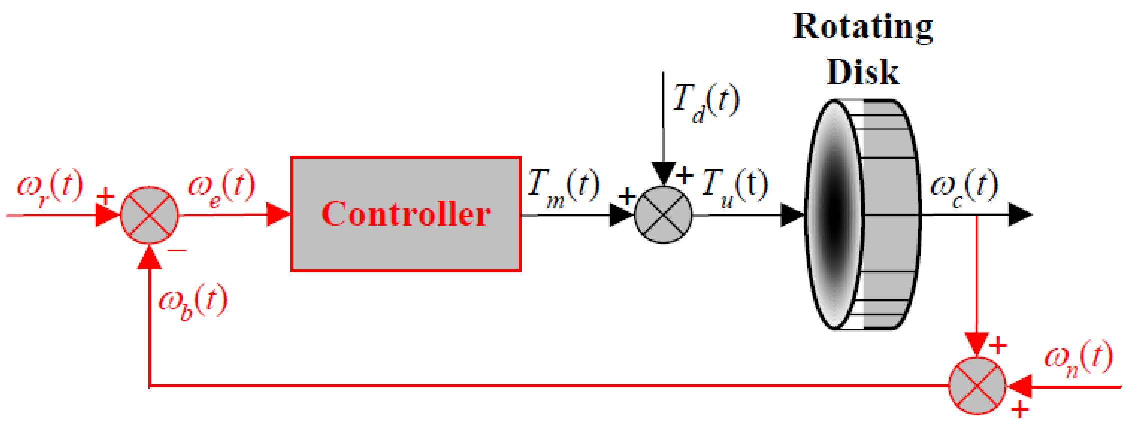

HOMEWORK #9

TIME-DOMAIN CONTROL

<u>Lag-Lead Speed Control of an Inertial Plant</u>

ME 411: System Modeling Analysis and Control

Fall 2010

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a) Block Diagram and transfer function:

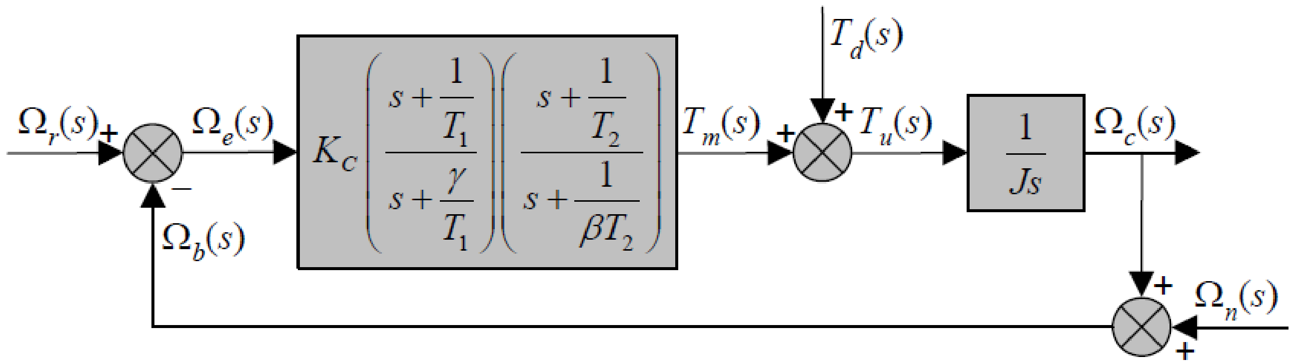

Figure 1.1: Lag-lead control system

$$G_c(s) = K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) \qquad G_p(s) = \frac{1}{Js}$$

From the above block diagram the equation we find are as follows:

$$\boldsymbol{\Omega_e(s) = \Omega_r(s) - \Omega_b(s)} \qquad \boldsymbol{\Omega_c(s) = T_u(s) * G_P(s)}$$

$$\boldsymbol{\Omega_b(s) = \Omega_c(s) + \Omega_n(s)} \qquad \boldsymbol{T_u(s) = T_m(s) + T_d(s)}$$

$$\boldsymbol{T_m(s) = G_c(s) * \Omega_e(s)}$$

(a) Transfer functions

(i) $\left.\frac{\Omega_c(s)}{\Omega_r(s)}\right|_{T_d(s)=\Omega_n(s)=0}$

which gives, $T_u(s) = T_m(s)$ and $\Omega_b(s) = \Omega_c(s)$

$$\Omega_c(s) = T_u(s) * G_P(s)$$

$$\Omega_c(s) = T_m(s) * G_P(s)$$

$$\Omega_c(s) = G_c(s) * \Omega_e(s) * G_P(s)$$

$$\Omega_c(s) = G_c(s) * G_P(s) * [\Omega_r(s) - \Omega_b(s)]$$

$$\Omega_c(s) = G_c(s) * G_P(s) * [\Omega_r(s) - \Omega_c(s)]$$

$$\Omega_c(s) = G_c(s) * G_P(s) * \Omega_r(s) - G_c(s) * G_P(s) * \Omega_c(s)$$

$$\Omega_c(s) + G_c(s) * G_P(s) * \Omega_c(s) = G_c(s) * G_P(s) * \Omega_r(s)$$

$$\Omega_c(s)[1 + G_c(s) * G_P(s)] = G_c(s) * G_P(s) * \Omega_r(s)$$

$$\frac{\Omega_c(s)}{\Omega_r(s)} = \frac{G_c(s) * G_P(s)}{[1 + G_c(s) * G_P(s)]}$$

$$\frac{\Omega_c(s)}{\Omega_r(s)} = \frac{K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) * \frac{1}{Js}}{\left[1 + K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) * \frac{1}{Js}\right]}$$

(ii) $\left.\boldsymbol{\frac{\Omega_c(s)}{T_d(s)}}\right|_{\boldsymbol{\Omega_n(s)=\Omega_r(s)=0}}$

Which gives, $\Omega_b(s) = \Omega_c(s)$ and $\Omega_e(s) = -\Omega_b(s)$

$$\Omega_c(s) = T_u(s) * G_P(s)$$

$$\Omega_c(s) = [T_m(s) + T_d(s)]G_P(s)$$

$$\Omega_c(s) = [G_c(s) * \Omega_e(s) + T_d(s)]G_P(s)$$

$$\Omega_c(s) = [G_c(s) * -\Omega_b(s) + T_d(s)]G_P(s)$$

$$\Omega_c(s) = [G_c(s) * -\Omega_c(s) + T_d(s)]G_P(s)$$

$$\Omega_c(s) = -\Omega_c(s) * G_c(s) * G_P(s) + T_d(s) * G_P(s)$$

$$\Omega_c(s) + \Omega_c(s) * G_c(s) * G_P(s) = T_d(s) * G_P(s)$$

$$\frac{\Omega_c(s)}{T_d(s)} = \frac{G_P(s)}{[1 + G_c(s) * G_P(s)]}$$

$$\frac{\Omega_c(s)}{T_d(s)} = \frac{\frac{1}{Js}}{\left[1 + K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) * \frac{1}{Js}\right]}$$

(iii) $\left.\frac{\boldsymbol{\Omega_c(s)}}{\boldsymbol{\Omega_n(s)}}\right|_{\boldsymbol{\Omega_r(s)=T_d(s)=0}}$

Which gives, $T_u(s) = T_m(s)$ and $\Omega_e(s) = -\Omega_b(s)$

$$\Omega_c(s) = T_u(s) * G_P(s)$$

$$\Omega_c(s) = T_m(s) * G_P(s)$$

$$\Omega_c(s) = G_c(s) * \Omega_e(s) * G_P(s)$$

$$\Omega_c(s) = -\Omega_b(s) * G_c(s) * G_P(s)$$

$$\Omega_c(s) = -[\Omega_c(s) + \Omega_n(s)]\, G_c(s) * G_P(s)$$

$$\Omega_c(s) = -\Omega_c(s) * G_c(s) * G_P(s) - \Omega_n(s) * G_c(s) * G_P(s)$$

$$\Omega_c(s) + \Omega_c(s) * G_c(s) * G_P(s) = -\Omega_n(s) * G_c(s) * G_P(s)$$

$$\frac{\Omega_c(s)}{\Omega_n(s)} = -\frac{G_c(s) * G_P(s)}{[1 + G_c(s) * G_P(s)]}$$

$$\frac{\Omega_c(s)}{\Omega_n(s)} = -\frac{K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) * \frac{1}{Js}}{\left[1 + K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) * \frac{1}{Js}\right]}$$

(iv) $\left.\frac{\boldsymbol{\Omega_e(s)}}{\boldsymbol{\Omega_r(s)}}\right|_{\boldsymbol{T_d(s)=\Omega_n(s)=0}}$

which, $T_u(s) = T_m(s)$ and $\Omega_b(s) = \Omega_c(s)$

$$\Omega_e(s) = \Omega_r(s) - \Omega_b(s)$$

$$\Omega_e(s) = \Omega_r(s) - \Omega_c(s)$$

$$\Omega_e(s) = \Omega_r(s) - [T_u(s) * G_P(s)]$$

$$\Omega_e(s) = \Omega_r(s) - [T_m(s) * G_P(s)]$$

$$\Omega_e(s) = \Omega_r(s) - [\Omega_e(s) * G_c(s) * G_P(s)]$$

$$\Omega_e(s) + \Omega_e(s) * G_c(s) * G_P(s) = \Omega_r(s)$$

$$\frac{\Omega_e(s)}{\Omega_r(s)} = \frac{1}{[1 + G_c(s) * G_P(s)]}$$

$$\frac{\Omega_e(s)}{\Omega_r(s)} = \frac{1}{\left[1 + K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) * \frac{1}{Js}\right]}$$

(v) $\left.\frac{\boldsymbol{\Omega}_{\mathbf{e}}(\boldsymbol{s})}{\boldsymbol{T}_{\boldsymbol{d}}(\boldsymbol{s})}\right|_{\boldsymbol{\Omega}_{\boldsymbol{n}}(\boldsymbol{s})=\boldsymbol{\Omega}_{\boldsymbol{r}}(\boldsymbol{s})=\mathbf{0}}$

Which gives $\Omega_{\mathrm{e}}(s) = -\Omega_{\mathrm{b}}(s)$ and $\Omega_b(s) = \Omega_{\mathrm{c}}(s)$

$$\Omega_{\mathrm{e}}(s) = \frac{T_m(s)}{G_c(s)}$$

$$\Omega_{\mathrm{e}}(s) = \frac{[T_u(s) - T_d(s)]}{G_c(s)}$$

$$\Omega_{\mathrm{e}}(s) = \frac{T_u(s)}{G_c(s)} - \frac{T_d(s)}{G_c(s)}$$

$$\Omega_{\mathrm{e}}(s) = \frac{\Omega_{\mathrm{c}}(s)}{G_c(s) * G_p(s)} - \frac{T_d(s)}{G_c(s)}$$

$$\Omega_{\mathrm{e}}(s) = \frac{\Omega_{\mathrm{b}}(s)}{G_c(s) * G_p(s)} - \frac{T_d(s)}{G_c(s)}$$

$$\Omega_{\mathrm{e}}(s) = \frac{-\Omega_{\mathrm{e}}(s)}{G_c(s) * G_p(s)} - \frac{T_d(s)}{G_c(s)}$$

$$\Omega_{\mathrm{e}}(s) + \frac{\Omega_{\mathrm{e}}(s)}{G_c(s) * G_p(s)} = -\frac{T_d(s)}{G_c(s)}$$

$$\frac{\left[G_c(s) * G_p(s)\right] \Omega_{\mathrm{e}}(s) + \Omega_{\mathrm{e}}(s)}{G_c(s) * G_p(s)} = -\frac{T_d(s)}{G_c(s)}$$

$$\Omega_{\mathrm{e}}(s) \frac{\left[G_c(s) * G_p(s) + 1\right]}{G_c(s) * G_p(s)} = -\frac{T_d(s)}{G_c(s)}$$

$$\frac{\Omega_{\mathrm{e}}(s)}{T_d(s)} = -\frac{\cancel{G_c(s)} * G_p(s)}{\cancel{G_c(s)}} * \frac{1}{\left[G_c(s) * G_p(s) + 1\right]}$$

$$\frac{\Omega_{\mathrm{e}}(s)}{T_d(s)} = -\frac{G_p(s)}{\left[G_c(s) * G_p(s) + 1\right]}$$

$$\frac{\Omega_{\mathrm{e}}(s)}{T_d(s)} = -\frac{\frac{1}{Js}}{\left[K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) * \frac{1}{Js} + 1\right]}$$

(vi) $\left.\frac{\boldsymbol{\Omega}_{\mathbf{e}}(\boldsymbol{s})}{\boldsymbol{\Omega}_{\mathbf{n}}(\boldsymbol{s})}\right|_{\boldsymbol{\Omega}_{\boldsymbol{r}}(\boldsymbol{s})=\boldsymbol{T}_{\boldsymbol{d}}(\boldsymbol{s})=\mathbf{0}}$

Which gives $T_u(s) = T_m(s)$ and $\Omega_e(s) = -\Omega_{\mathrm{b}}(s)$

$$\Omega_{\mathrm{e}}(s) = \frac{T_m(s)}{G_c(s)}$$

$$\Omega_{\mathrm{e}}(s) = \frac{T_u(s)}{G_c(s)}$$

$$\Omega_{\mathrm{e}}(s) = \frac{\Omega_{\mathrm{c}}(s)}{G_c(s)G_p(s)}$$

$$\Omega_{\mathrm{e}}(s) = \frac{\Omega_{\mathrm{b}}(s) - \Omega_{\mathrm{n}}(s)}{G_c(s)G_p(s)}$$

$$\Omega_{\mathrm{e}}(s) = \frac{-\Omega_{\mathrm{e}}(s) - \Omega_{\mathrm{n}}(s)}{G_c(s)G_p(s)}$$

$$\Omega_{\mathrm{e}}(s) = -\frac{\Omega_{\mathrm{e}}(s)}{G_c(s)G_p(s)} - \frac{\Omega_{\mathrm{n}}(s)}{G_c(s)G_p(s)}$$

$$\Omega_{\mathrm{e}}(s) + \frac{\Omega_{\mathrm{e}}(s)}{G_c(s)G_p(s)} = -\frac{\Omega_{\mathrm{n}}(s)}{G_c(s)G_p(s)}$$

$$\Omega_{\mathrm{e}}(s)\left[\frac{1+G_c(s)G_p(s)}{G_c(s)G_p(s)}\right] = -\frac{\Omega_{\mathrm{n}}(s)}{G_c(s)G_p(s)}$$

$$\frac{\Omega_{\mathrm{e}}(s)}{\Omega_{\mathrm{n}}(s)} = -\frac{\cancel{G_c(s)G_p(s)}}{\cancel{G_c(s)G_p(s)}} * \frac{1}{1+G_c(s)G_p(s)}$$

$$\frac{\Omega_{\mathrm{e}}(s)}{\Omega_{\mathrm{n}}(s)} = -\frac{1}{1 + G_c(s)G_p(s)}$$

$$\frac{\Omega_{\mathrm{e}}(s)}{\Omega_{\mathrm{n}}(s)} = -\frac{1}{1 + K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)\frac{1}{Js}}$$

(b) **Open –loop gains and system types:**

Given:

$$L(s) = G_c(s)G_P(s)H(s) = K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)\frac{1}{Js}\cdot 1$$

$$L(s) = \frac{K_C}{Js}\left(\frac{sT_1+1}{sT_1+\gamma}\right)\left(\frac{s\beta T_2+\beta}{s\beta T_2+1}\right) = \left[\frac{K_C sT_1 + K_C}{Js^2T_1 + Js\gamma}\right] * \left[\frac{s\beta T_2+\beta}{s\beta T_2+1}\right]$$

$$L(s) = \left[\frac{K_C s^2 T_1\beta T_2 + K_C sT_1\beta + K_C s\beta T_2 + K_C\beta}{Js^3T_1T_2\beta + Js^2T_1 + Js^2\gamma\beta T_2 + Js\gamma}\right]$$

$$Therefore\ , the\ open\ loop\ gain\ is: L(s) = \frac{K_C\beta}{Js}\left[\frac{s^2T_1T_2 + sT_1 + sT_2 + 1}{s^2T_1T_2\beta + sT_1 + s\gamma\beta T_2 + \gamma}\right]$$

$$\frac{\Omega_c(s)}{\Omega_r(s)} = \frac{L(s)}{1+L(s)}$$

Steady-State Error:

$$E(s) = 1 - \frac{\Omega_c(s)}{\Omega_r(s)}$$

Using the FVT, the steady-state error is given by:

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\left(1 - \frac{\Omega_c(s)}{\Omega_r(s)}\right)$$

$$= \lim_{s\to 0} s\left(1 - \frac{L(s)}{1+L(s)}\right)R(s)$$

$$= \lim_{s\to 0} s\left(\frac{1+L(s)-L(s)}{1+L(s)}\right)R(s)$$

$$= \lim_{s\to 0} s\left(\frac{1}{1+L(s)}\right)R(s)$$

Consider a polynomial input: $R(s) = \frac{1}{s^k}$

$$e_{ss} = \lim_{s\to 0}\frac{s}{s^k}\left(\frac{1}{1+\frac{K_C\beta}{Js}\left[\frac{s^2T_1T_2+sT_1+sT_2+1}{s^2T_1T_2\beta+sT_1+s\gamma\beta T_2+\gamma}\right]}\right)$$

k	e_{ss}
0	0
1	0
2	infinity

Therefore the system is type 1 because a *unity feedback system* is defined to be type k if the feedback system guarantees.

$$e_{ss} = 0 \quad for \quad R(s) = \frac{1}{s^k}$$

$$|e_{ss}| < \infty \quad for \quad R(s) = \frac{1}{s^{k+1}}$$

(c) Steady-states and static-error constants:

- ***Step Input*:**

Find K_P, $\omega_{c,s.s}|_{\mathrm{T_d(t)=\Omega_n(t)=0}}^{\mathrm{\omega_r(t)=\bar{\omega}_r \cdot 1(t)}}$ and $\omega_{e,s.s}|_{\mathrm{T_d(t)=\Omega_n(t)=0}}^{\mathrm{\omega_r(t)=\bar{\omega}_r \cdot 1(t)}}$

From the table, for step input we have, the reference angular speed $\mathrm{\omega_r(t) = \bar{\omega}_r \cdot 1(t)}$, the Disturbance Torque, $\mathrm{T_d(t) = 0}$ and Noise angular speed $\mathrm{\omega_n(t) = 0}$.

Therefore,

$$K_P = \lim_{\mathrm{s \to 0}} \mathrm{L(s)} = \lim_{\mathrm{s \to 0}} \frac{K_C \beta}{Js} \left[\frac{s^2 T_1 T_2 + sT_1 + sT_2 + 1}{s^2 T_1 T_2 \beta + sT_1 + s\gamma\beta T_2 + \gamma} \right] = \infty$$

$$\omega_{c,s.s}|_{\mathrm{T_d(t)=\Omega_n(t)=0}}^{\mathrm{\omega_r(t)=\bar{\omega}_r \cdot 1(t)}} = \lim_{t \to \infty} \omega_c(t)|_{\mathrm{T_d(t)=\omega_n(t)=0}}^{\mathrm{\omega_r(t)=\bar{\omega}_r \cdot 1(t)}}$$

$$= \lim_{\mathrm{s \to 0}} \left[\mathrm{s} \cdot \ \Omega_c(\mathrm{s})|_{\mathrm{T_d(t)=\ \Omega_n(s)=0}}^{\mathrm{\omega_r(t)=\frac{\bar{\omega}_r}{s}}} \right] = \lim_{\mathrm{s \to 0}} \left[\cancel{\mathrm{s}} \cdot \frac{\Omega_c(\mathrm{s})}{\Omega_r(\mathrm{s})} |_{\mathrm{T_d(t)=\ \Omega_n(s)=0}} \cdot \frac{\bar{\omega}_r}{\cancel{\mathrm{s}}} \right]$$

$$= \lim_{\mathrm{s \to 0}} \left[\frac{\frac{K_C \beta}{Js} \left[\frac{s^2 T_1 T_2 + sT_1 + sT_2 + 1}{s^2 T_1 T_2 \beta + sT_1 + s\gamma\beta T_2 + \gamma} \right]}{1 + \frac{K_C \beta}{Js} \left[\frac{s^2 T_1 T_2 + sT_1 + sT_2 + 1}{s^2 T_1 T_2 \beta + sT_1 + s\gamma\beta T_2 + \gamma} \right]} \cdot \bar{\omega}_r \right]$$

$$\omega_{c,s.s}|_{\mathrm{T_d(t)=\Omega_n(t)=0}}^{\mathrm{\omega_r(t)=\bar{\omega}_r \cdot 1(t)}} = \bar{\omega}_r$$

$$\omega_{e,s.s}|_{\mathrm{T_d(t)=\omega_n(t)=0}}^{\mathrm{\omega_r(t)=\bar{\omega}_r \cdot 1(t)}} = \lim\nolimits_{t \to \infty} \omega_e(t)|_{\mathrm{T_d(t)=\omega_n(t)=0}}^{\mathrm{\omega_r(t)=\bar{\omega}_r \cdot 1(t)}}$$

$$= \lim_{\mathrm{s \to 0}} \left[\mathrm{s} \cdot \ \Omega_e(\mathrm{s})|_{\mathrm{T_d(t)=\ \Omega_n(s)=0}}^{\mathrm{\Omega_r(t)=\frac{\bar{\omega}_r}{s}}} \right] = \lim_{\mathrm{s \to 0}} \left[\cancel{\mathrm{s}} \cdot \frac{\Omega_e(\mathrm{s})}{\Omega_r(\mathrm{s})} |_{\mathrm{T_d(t)=\ \Omega_n(s)=0}} \cdot \frac{\bar{\omega}_r}{\cancel{\mathrm{s}}} \right]$$

$$= \lim_{s \to 0}\left[\frac{1}{\left[1 + K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) * \frac{1}{Js}\right]} \cdot \overline{\omega}_r\right]$$

$$\omega_{e,s.s}\Big|_{T_d(t)=\omega_n(t)=0}^{\omega_r(t)=\overline{\omega}_r \cdot 1(t)} = 0$$

Find $\omega_{c,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\overline{T}_d \cdot 1(t)}$ and $\omega_{e,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\overline{T}_d \cdot 1(t)}$

From the table, for step input we have, the reference angular speed $\omega_r(t) = 0$, the Disturbance Torque, $T_d(t) = \overline{T}_d \cdot 1(t) \rightarrow T_d(s) = \frac{\overline{T}_d}{s}$ and Noise angular speed $\omega_n(t) = 0$.

$$\omega_{c,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\overline{T}_d \cdot 1(t)} = \lim_{t \to \infty} \omega_c(t)\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\overline{T}_d \cdot 1(t)}$$

$$= \lim_{s \to 0}\left[s \cdot \Omega_c(s)\Big|_{\Omega_n(s)=\Omega_r(s)=0}^{T_d(s)=\frac{\overline{T}_d}{s}}\right] = \lim_{s \to 0}\left[\cancel{s} \cdot \frac{\Omega_c(s)}{T_d(s)}\Big|_{\Omega_n(s)=\Omega_r(s)=0} \cdot \frac{\overline{T}_d}{\cancel{s}}\right]$$

$$= \lim_{s \to 0}\left[\frac{\frac{1}{Js}}{\left[1 + K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) * \frac{1}{Js}\right]} \cdot \overline{T}_d\right]$$

$$\omega_{c,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\overline{T}_d \cdot 1(t)} = \frac{\gamma \overline{T}_d}{K_c \beta}$$

$$\omega_{e,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\overline{T}_d \cdot 1(t)} = \lim_{t \to \infty} \omega_e(t)\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\overline{T}_d \cdot 1(t)}$$

$$= \lim_{s \to 0}\left[s \cdot \Omega_e(s)\Big|_{\Omega_n(s)=\Omega_r(s)=0}^{T_d(s)=\frac{\overline{T}_d}{s}}\right] = \lim_{s \to 0}\left[\cancel{s} \cdot \frac{\Omega_e(s)}{T_d(s)}\Big|_{\Omega_n(s)=\Omega_r(s)=0} \cdot \frac{\overline{T}_d}{\cancel{s}}\right]$$

$$= \lim_{s \to 0}\left[-\frac{\frac{1}{Js}}{\left[K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) * \frac{1}{Js} + 1\right]} \cdot \overline{T}_d\right]$$

$$\omega_{e,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\overline{T}_d \cdot 1(t)} = -\frac{\gamma \overline{T}_d}{K_c \beta}$$

Find $\omega_{c,s.s}\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\bar{\omega}_n\cdot 1(t)}$ and $\omega_{e,s.s}\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\bar{\omega}_n\cdot 1(t)}$

From the table, for step input we have, the reference angular speed$\omega_r(t) = 0$, the Disturbance Torque, $T_d(t) = 0$ and Noise angular speed $\omega_n(t) = \bar{\omega}_n \cdot 1(t) \rightarrow \frac{\bar{\omega}_n}{s}$

$$\omega_{c,s.s}\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\bar{\omega}_n\cdot 1(t)} = \lim_{t\to\infty} \omega_c(t)\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\bar{\omega}_n\cdot 1(t)}$$

$$= \lim_{s\to 0}\left[s\cdot\ \Omega_c(s)\Big|_{\Omega_r(s)=T_d(s)=0}^{\omega_n(s)=\frac{\bar{\omega}_n}{s}}\right] = \lim_{s\to 0}\left[\cancel{s}\cdot\frac{\Omega_c(s)}{\Omega_n(s)}\Big|_{\Omega_r(s)=T_d(s)=0}\cdot\frac{\bar{\omega}_n}{\cancel{s}}\right]$$

$$= \lim_{s\to 0}\left[-\frac{K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) * \frac{1}{Js}}{\left[1+K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) * \frac{1}{Js}\right]}\cdot\bar{\omega}_n\right]$$

$$\omega_{c,s.s}\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\bar{\omega}_n\cdot 1(t)} = -\bar{\omega}_n$$

$$\omega_{e,s.s}\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\bar{\omega}_n\cdot 1(t)} = \lim_{t\to\infty} \omega_e(t)\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\bar{\omega}_n\cdot 1(t)}$$

$$= \lim_{s\to 0}\left[s\cdot\ \Omega_e(s)\Big|_{\Omega_r(s)=T_d(s)=0}^{\omega_n(s)=\frac{\bar{\omega}_n}{s}}\right] = \lim_{s\to 0}\left[\cancel{s}\cdot\frac{\Omega_e(s)}{\Omega_n(s)}\Big|_{\Omega_r(s)=T_d(s)=0}\cdot\frac{\bar{\omega}_n}{\cancel{s}}\right]$$

$$= \lim_{s\to 0}\left[-\frac{1}{1+K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)\frac{1}{Js}}\cdot\bar{\omega}_n\right]$$

$$\omega_{e,s.s}\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\bar{\omega}_n\cdot 1(t)} = 0$$

- ***Ramp Input***:

Find K_v, $\omega_{c,s.s}\Big|_{T_d(t)=\omega_n(t)=0}^{\omega_r(t)=\omega'_r\, t\cdot 1(t)}$ and $\omega_{e,s.s}\Big|_{T_d(t)=\Omega_n(t)=0}^{\omega_r(t)=\bar{\omega}_r\cdot t.1(t)}$

From the table, for step input we have, the reference angular speed $\omega_r(t) = \omega'_r\, t\cdot 1(t) \rightarrow \frac{\omega'_r}{s^2}$, the Disturbance Torque, $T_d(t) = 0$ and Noise angular speed $\omega_n(t) = 0$.

Therefore,

$$K_v = \lim_{s\to 0} s L(s) = \lim_{s\to 0} \cancel{s}\frac{K_C\beta}{J\cancel{s}}\left[\frac{s^2T_1T_2 + sT_1 + sT_2 + 1}{s^2T_1T_2\beta + sT_1 + s\gamma\beta T_2 + \gamma}\right] = \frac{K_C\beta}{J\gamma}$$

$$\omega_{c,s.s}\Big|_{\mathrm{T_d(t)=\Omega_n(t)=0}}^{\mathrm{\omega_r(t)=\omega'_r\, t\cdot 1(t)}} = \lim_{t\to\infty}\omega_c(t)\Big|_{\mathrm{T_d(t)=\omega_n(t)=0}}^{\mathrm{\omega_r(t)=\omega'_r\, t\cdot 1(t)}}$$

$$= \lim_{s\to 0}\left[s\cdot\ \Omega_c(s)\Big|_{\mathrm{T_d(t)=\ \Omega_n(s)=0}}^{\mathrm{\omega_r(t)=\frac{\omega'_r}{s^2}}}\right] = \lim_{s\to 0}\left[\cancel{s}\cdot\frac{\Omega_c(s)}{\Omega_r(s)}\Big|_{\mathrm{T_d(t)=\ \Omega_n(s)=0}}\cdot\frac{\omega'_r}{\cancel{s^2}}\right]$$

$$= \lim_{s\to 0}\left[\frac{\frac{K_C\beta}{Js}\left[\frac{s^2T_1T_2+sT_1+sT_2+1}{s^2T_1T_2\beta+sT_1+s\gamma\beta T_2+\gamma}\right]}{1+\frac{K_C\beta}{Js}\left[\frac{s^2T_1T_2+sT_1+sT_2+1}{s^2T_1T_2\beta+sT_1+s\gamma\beta T_2+\gamma}\right]}\cdot\frac{\omega'_r}{s}\right]$$

$$\omega_{c,s.s}\Big|_{\mathrm{T_d(t)=\Omega_n(t)=0}}^{\mathrm{\omega_r(t)=\omega'_r\, t\cdot 1(t)}} = infinity$$

$$\omega_{e,s.s}\Big|_{\mathrm{T_d(t)=\omega_n(t)=0}}^{\mathrm{\omega_r(t)=\omega'_r\, t\cdot 1(t)}} = \lim_{t\to\infty}\omega_e(t)\Big|_{\mathrm{T_d(t)=\omega_n(t)=0}}^{\mathrm{\omega_r(t)=\omega'_r\, t\cdot 1(t)}}$$

$$= \lim_{s\to 0}\left[s\cdot\ \Omega_e(s)\Big|_{\mathrm{T_d(t)=\ \Omega_n(s)=0}}^{\mathrm{\Omega_r(t)=\frac{\omega'_r}{s^2}}}\right] = \lim_{s\to 0}\left[\cancel{s}\cdot\frac{\Omega_e(s)}{\Omega_r(s)}\Big|_{\mathrm{T_d(t)=\ \Omega_n(s)=0}}\cdot\frac{\omega'_r}{\cancel{s^2}}\right]$$

$$= \lim_{s\to 0}\left[\frac{1}{\left[1+K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)*\frac{1}{Js}\right]}\cdot\frac{\omega'_r}{s}\right]$$

$$\omega_{e,s.s}\Big|_{\mathrm{T_d(t)=\omega_n(t)=0}}^{\mathrm{\omega_r(t)=\omega'_r\, t\cdot 1(t)}} = 0$$

Find $\omega_{c,s.s}\Big|_{\mathrm{\omega_n(t)=\omega_r(t)=0}}^{\mathrm{T_d(t)=T'_d\, t\cdot 1(t)}}$ and $\omega_{e,s.s}\Big|_{\mathrm{\omega_n(t)=\omega_r(t)=0}}^{\mathrm{T_d(t)=T'_d\, t\cdot 1(t)}}$

From the table, for step input we have, the reference angular speed $\mathrm{\omega_r(t)} = 0$, the Disturbance Torque, $\mathrm{T_d(t) = T'_d\ \cdot 1(t)} \to \frac{\mathrm{T'_d}}{s^2}$ and Noise angular speed $\mathrm{\omega_n(t)} = 0$.

$$\omega_{c,s.s}\Big|_{\mathrm{\omega_n(t)=\omega_r(t)=0}}^{\mathrm{T_d(t)=T'_d\, t\cdot 1(t)}} = \lim_{t\to\infty}\omega_c(t)\Big|_{\mathrm{\omega_n(t)=\omega_r(t)=0}}^{\mathrm{T_d(t)=T'_d\, t\cdot 1(t)}}$$

$$= \lim_{s\to 0}\left[s\cdot\ \Omega_c(s)\Big|_{\mathrm{\Omega_n(s)=\ \Omega_r(s)=0}}^{\mathrm{T_d(s)=\frac{T'_d}{s^2}}}\right] = \lim_{s\to 0}\left[\cancel{s}\cdot\frac{\Omega_c(s)}{\mathrm{T_d(s)}}\Big|_{\mathrm{\Omega_n(s)=\ \Omega_r(s)=0}}\cdot\frac{\mathrm{T'_d}}{s^2}\right]$$

$$= \lim_{s\to 0}\left[\frac{\frac{1}{Js}}{\left[1+K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)*\frac{1}{Js}\right]}\cdot\frac{\mathrm{T'_d}}{s}\right]$$

$$\omega_{c,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=T'_d\, t\cdot 1(t)} = \text{infinity}$$

$$\omega_{e,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=T'_d\, t\cdot 1(t)} = \lim_{t\to\infty}\omega_e(t)\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=T'_d\, t\cdot 1(t)}$$

$$= \lim_{s\to 0}\left[s\cdot\ \Omega_e(s)\Big|_{\Omega_n(s)=\ \Omega_r(s)=0}^{T_d(s)=\frac{T'_d}{s^2}}\right] = \lim_{s\to 0}\left[\cancel{s}\cdot\frac{\Omega_e(s)}{T_d(s)}\Big|_{\Omega_n(s)=\ \Omega_r(s)=0}\cdot\frac{T'_d}{\cancel{s^2}}\right]$$

$$= \lim_{s\to 0}\left[-\frac{\frac{1}{Js}}{\left[K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)*\frac{1}{Js}+1\right]}\cdot\frac{T'_d}{s}\right]$$

$$\omega_{e,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=T'_d\, t\cdot 1(t)} = -\text{infinity}$$

Find $\omega_{c,s.s}\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\omega'_n t\cdot 1(t)}$ and $\omega_{e,s.s}\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\omega'_n t\cdot 1(t)}$

From the table, for step input we have, the reference angular speed$\omega_r(t) = 0$, the Disturbance Torque, $T_d(t) = 0$ and Noise angular speed $\omega_n(t) = \bar{\omega}_n\cdot 1(t) \to \frac{\omega'_n}{s^2}$

$$\omega_{c,s.s}\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\omega'_n t\cdot 1(t)} = \lim_{t\to\infty}\omega_c(t)\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\omega'_n t\cdot 1(t)}$$

$$= \lim_{s\to 0}\left[s\cdot\ \Omega_c(s)\Big|_{\Omega_r(s)=T_d(s)=0}^{\omega_n(s)=\frac{\omega'_n}{s^2}}\right] = \lim_{s\to 0}\left[\cancel{s}\cdot\frac{\Omega_c(s)}{\Omega_n(s)}\Big|_{\Omega_r(s)=T_d(s)=0}\cdot\frac{\omega'_n}{\cancel{s^2}}\right]$$

$$= \lim_{s\to 0}\left[-\frac{K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)*\frac{1}{Js}}{\left[1+K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)*\frac{1}{Js}\right]}\cdot\frac{\omega'_n}{s}\right]$$

$$\omega_{c,s.s}\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\omega'_n t\cdot 1(t)} = -\text{infinity}$$

$$\omega_{e,s.s}\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\omega'_n t\cdot 1(t)} = \lim_{t\to\infty}\omega_e(t)\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\omega'_n t\cdot 1(t)}$$

$$= \lim_{s\to 0}\left[s \cdot \Omega_e(s)\Big|_{\Omega_r(s)=T_d(s)=0}^{\omega_n(s)=\frac{\omega'_n}{s^2}} \right] = \lim_{s\to 0}\left[\cancel{s} \cdot \frac{\Omega_e(s)}{\Omega_n(s)}\Big|_{\Omega_r(s)=T_d(s)=0} \cdot \frac{\omega'_n}{\cancel{s^2}} \right]$$

$$= \lim_{s\to 0}\left[-\frac{1}{1 + K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)\frac{1}{Js}} \cdot \frac{\omega'_n}{s} \right]$$

$$\omega_{c,s.s}\Big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\omega'_n t\cdot 1(t)} = -\frac{J\gamma\omega'_n}{K_c\beta}$$

- ***Parabolic Input***:

Find K_a, $\omega_{c,s.s}\Big|_{T_d(t)=\omega_n(t)=0}^{\omega_r(t)=\breve{\omega}_r\frac{t^2}{2}\cdot 1(t)}$ and $\omega_{e,s.s}\Big|_{T_d(t)=\Omega_n(t)=0}^{\omega_r(t)=\breve{\omega}_r\frac{t^2}{2}\cdot 1(t)}$

From the table, for step input we have, the reference angular speed $\omega_r(t) = \breve{\omega}_r\,\frac{t^2}{2}\cdot 1(t) \to \frac{\breve{\omega}_r}{s^3}$, the Disturbance Torque, $T_d(t) = 0$ and Noise angular speed $\omega_n(t) = 0$.

Therefore,

$$K_a = \lim_{s\to 0} s^2 L(s) = \lim_{s\to 0} \cancel{s^2}\frac{K_C\beta}{J\cancel{s}}\left[\frac{s^2T_1T_2 + sT_1 + sT_2 + 1}{s^2T_1T_2\beta + sT_1 + s\gamma\beta T_2 + \gamma}\right] = 0$$

$$\omega_{c,s.s}\Big|_{T_d(t)=\Omega_n(t)=0}^{\omega_r(t)=\breve{\omega}_r\frac{t^2}{2}\cdot 1(t)} = \lim_{t\to\infty} \omega_c(t)\Big|_{T_d(t)=\omega_n(t)=0}^{\omega_r(t)=\breve{\omega}_r\frac{t^2}{2}\cdot 1(t)}$$

$$= \lim_{s\to 0}\left[s \cdot \Omega_c(s)\Big|_{T_d(t)=\Omega_n(s)=0}^{\omega_r(t)=\frac{\breve{\omega}_r}{s^3}} \right] = \lim_{s\to 0}\left[\cancel{s} \cdot \frac{\Omega_c(s)}{\Omega_r(s)}\Big|_{T_d(t)=\Omega_n(s)=0} \cdot \frac{\breve{\omega}_r}{\cancel{s^3}} \right]$$

$$= \lim_{s\to 0}\left[\frac{\frac{K_C\beta}{Js}\left[\frac{s^2T_1T_2+sT_1+sT_2+1}{s^2T_1T_2\beta+sT_1+s\gamma\beta T_2+\gamma}\right]}{1 + \frac{K_C\beta}{Js}\left[\frac{s^2T_1T_2+sT_1+sT_2+1}{s^2T_1T_2\beta+sT_1+s\gamma\beta T_2+\gamma}\right]} \cdot \frac{\breve{\omega}_r}{s^2} \right]$$

$$\omega_{c,s.s}\Big|_{T_d(t)=\Omega_n(t)=0}^{\omega_r(t)=\breve{\omega}_r\frac{t^2}{2}\cdot 1(t)} = infinity$$

$$\omega_{e,s.s}\Big|_{T_d(t)=\omega_n(t)=0}^{\omega_r(t)=\breve{\omega}_r\frac{t^2}{2}\cdot 1(t)} = \lim_{t\to\infty} \omega_e(t)\Big|_{T_d(t)=\omega_n(t)=0}^{\omega_r(t)=\breve{\omega}_r\frac{t^2}{2}\cdot 1(t)}$$

$$= \lim_{s\to 0}\left[s \cdot \Omega_e(s)\Big|_{T_d(t)=\Omega_n(s)=0}^{\Omega_r(t)=\frac{\breve{\omega}_r}{s^3}} \right] = \lim_{s\to 0}\left[\cancel{s} \cdot \frac{\Omega_e(s)}{\Omega_r(s)}\Big|_{T_d(t)=\Omega_n(s)=0} \cdot \frac{\breve{\omega}_r}{\cancel{s^3}} \right]$$

$$= \lim_{s\to 0}\left[\frac{1}{\left[1+K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)*\frac{1}{Js}\right]}\cdot\frac{\breve{\omega}_r}{s^2}\right]$$

$$\omega_{e,s.s}\Big|_{T_d(t)=\omega_n(t)=0}^{\omega_r(t)=\breve{\omega}_r\frac{t^2}{2}\cdot 1(t)} = infinity$$

Find $\omega_{c,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\breve{T}_d\frac{t^2}{2}\cdot 1(t)}$ and $\omega_{e,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\breve{T}_d\frac{t^2}{2}\cdot 1(t)}$

From the table, for step input we have, the reference angular speed $\omega_r(t) = 0$, the Disturbance Torque, $T_d(t) = \breve{T}_d\,\frac{t^2}{2}\cdot 1(t) \to \frac{T'_d}{s^3}$ and Noise angular speed $\omega_n(t) = 0$.

$$\omega_{c,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\breve{T}_d\frac{t^2}{2}\cdot 1(t)} = \lim_{t\to\infty}\omega_c(t)\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\breve{T}_d\frac{t^2}{2}\cdot 1(t)}$$

$$= \lim_{s\to 0}\left[s\cdot\ \Omega_c(s)\Big|_{\Omega_n(s)=\Omega_r(s)=0}^{T_d(s)=\frac{\breve{T}_d}{s^3}}\right] = \lim_{s\to 0}\left[\cancel{s}\cdot\frac{\Omega_c(s)}{T_d(s)}\Big|_{\Omega_n(s)=\Omega_r(s)=0}\cdot\frac{\breve{T}_d}{\cancel{s^3}}\right]$$

$$= \lim_{s\to 0}\left[\frac{\frac{1}{Js}}{\left[1+K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)*\frac{1}{Js}\right]}\cdot\frac{\breve{T}_d}{s^2}\right]$$

$$\omega_{c,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\breve{T}_d\frac{t^2}{2}\cdot 1(t)} = \text{infinity}$$

$$\omega_{e,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\breve{T}_d\frac{t^2}{2}\cdot 1(t)} = \lim_{t\to\infty}\omega_e(t)\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\breve{T}_d\frac{t^2}{2}\cdot 1(t)}$$

$$= \lim_{s\to 0}\left[s\cdot\ \Omega_e(s)\Big|_{\Omega_n(s)=\Omega_r(s)=0}^{T_d(s)=\frac{\breve{T}_d}{s^3}}\right] = \lim_{s\to 0}\left[\cancel{s}\cdot\frac{\Omega_e(s)}{T_d(s)}\Big|_{\Omega_n(s)=\Omega_r(s)=0}\cdot\frac{\breve{T}_d}{\cancel{s^3}}\right]$$

$$= \lim_{s\to 0}\left[-\frac{\frac{1}{Js}}{\left[K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)*\frac{1}{Js}+1\right]}\cdot\frac{\breve{T}_d}{s^2}\right]$$

$$\omega_{e,s.s}\Big|_{\omega_n(t)=\omega_r(t)=0}^{T_d(t)=\breve{T}_d\frac{t^2}{2}\cdot 1(t)} = -\text{infinity}$$

Find $\omega_{c,s.s}\big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\breve{\omega}_n\frac{t^2}{2}\cdot 1(t)}$ and $\omega_{e,s.s}\big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\breve{\omega}_n\frac{t^2}{2}\cdot 1(t)}$

From the table, for step input we have, the reference angular speed$\omega_r(t) = 0$, the Disturbance Torque, $T_d(t) = 0$ and Noise angular speed $\omega_n(t) = \breve{\omega}_n \frac{t^2}{2} \cdot 1(t) \rightarrow \frac{\breve{\omega}_n}{s^3}$

$$\omega_{c,s.s}\big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\breve{\omega}_n\frac{t^2}{2}\cdot 1(t)} = \lim_{t\to\infty} \omega_c(t)\big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\breve{\omega}_n\frac{t^2}{2}\cdot 1(t)}$$

$$= \lim_{s\to 0}\left[s\cdot\ \Omega_c(s)\big|_{\Omega_r(s)=T_d(s)=0}^{\omega_n(s)=\frac{\breve{\omega}_n}{s^3}}\right] = \lim_{s\to 0}\left[\cancel{s}\cdot\frac{\Omega_c(s)}{\Omega_n(s)}\big|_{\Omega_r(s)=T_d(s)=0}\cdot\frac{\breve{\omega}_n}{\cancel{s^3}}\right]$$

$$= \lim_{s\to 0}\left[-\frac{K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) * \frac{1}{Js}}{\left[1+K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) * \frac{1}{Js}\right]}\cdot\frac{\breve{\omega}_n}{s^2}\right]$$

$$\omega_{c,s.s}\big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\breve{\omega}_n\frac{t^2}{2}\cdot 1(t)} = -\text{infinity}$$

$$\omega_{e,s.s}\big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\breve{\omega}_n\frac{t^2}{2}\cdot 1(t)} = \lim_{t\to\infty} \omega_e(t)\big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\breve{\omega}_n\frac{t^2}{2}\cdot 1(t)}$$

$$= \lim_{s\to 0}\left[s\cdot\ \Omega_e(s)\big|_{\Omega_r(s)=T_d(s)=0}^{\omega_n(s)=\frac{\breve{\omega}_n}{s^3}}\right] = \lim_{s\to 0}\left[\cancel{s}\cdot\frac{\Omega_e(s)}{\Omega_n(s)}\big|_{\Omega_r(s)=T_d(s)=0}\cdot\frac{\breve{\omega}_n}{\cancel{s^3}}\right]$$

$$= \lim_{s\to 0}\left[-\frac{1}{1+K_C\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)\frac{1}{Js}}\cdot\frac{\breve{\omega}_n}{s^2}\right]$$

$$\omega_{c,s.s}\big|_{\omega_r(t)=T_d(t)=0}^{\omega_n(t)=\breve{\omega}_n\frac{t^2}{2}\cdot 1(t)} = \text{infinity}$$

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Lag-Lead Speed Control of an Inertial Plant

Plant Dynamics	Controller	Open-Loop Gain	System Type
$G_p(s)=\dfrac{1}{Js}$	$G_c(s)=K_C\left(\dfrac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\dfrac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)$	$L(s)=\dfrac{K_C\beta}{Js}\left[\dfrac{s^2T_1T_2+sT_1+sT_2+1}{s^2T_1T_2\beta+sT_1+s\gamma\beta T_2+\gamma}\right]$	1

Input Type	Reference Ang. Speed, $\omega_r(t)$	Disturbance Torque, $T_d(t)$	Noise Ang. Speed, $\omega_n(t)$	Static Error Constant: Type	Static Error Constant: Expression	Steady-State Output $\omega_{c,s.s.}=\lim_{t\to\infty}\omega_c(t)$	Steady-State Error $\omega_{e,s.s.}=\lim_{t\to\infty}\omega_e(t)$
Step	$\bar{\omega}_r\cdot 1(t)$	0	0	K_p	∞	$\overline{\omega_r}$	0
	0	$\bar{T}_d\cdot 1(t)$	0			$\gamma\overline{\overline{T_d}}/K_c\beta$	$-\gamma\overline{\overline{T_d}}/K_c\beta$
	0	0	$\bar{\omega}_n\cdot 1(t)$			$-\overline{\omega_n}$	$Zero$
Ramp	$\omega_r' t\cdot 1(t)$	0	0	K_v	$\dfrac{K_c\beta}{J\gamma}$	∞	0
	0	$T_d' t\cdot 1(t)$	0			∞	$-\infty$
	0	0	$\omega_n' t\cdot 1(t)$			$-\infty$	$J\gamma\omega_n'/K_c\beta$
Parabolic	$\breve{\omega}_r\frac{t^2}{2}\cdot 1(t)$	0	0	K_a	0	∞	∞
	0	$\breve{T}_d\frac{t^2}{2}\cdot 1(t)$	0			∞	$-\infty$
	0	0	$\breve{\omega}_n\frac{t^2}{2}\cdot 1(t)$			$-\infty$	∞

ME 41100: SYSTEMS MODELING, ANALYSIS AND CONTROL	HOMEWORK #9: TIME-DOMAIN CONTROL

(d) Routh-Hurwitz stability criterion:

Routh-Hurwitz stability criterion Table			
s^3	$a_0 = JT_1T_2\beta$	$a_2 = J\gamma + K_C\beta T_1 + K_C\beta T_2$	$a_4 = 0$
s^2	$a_1 = JT_1 + J\gamma\beta T_2 + K_C\beta T_1 T_2$	$a_3 = K_C\beta$	$a_5 = 0$
s^1	$b_1 = \dfrac{a_1a_2 - a_0a_3}{a_1}$	$b_2 = \dfrac{a_1a_4 - a_0a_5}{a_1} = 0$	$b_3 = \dfrac{a_1a_6 - a_0a_7}{a_1} = 0$
s^0	$c_1 = \dfrac{b_1a_3 - a_1b_2}{b_1} = a_3 = K_C\beta$	$c_2 = \dfrac{b_1a_5 - a_1b_3}{b_1} = 0$	$c_3 = \dfrac{b_1a_7 - a_1b_4}{b_1} = 0$

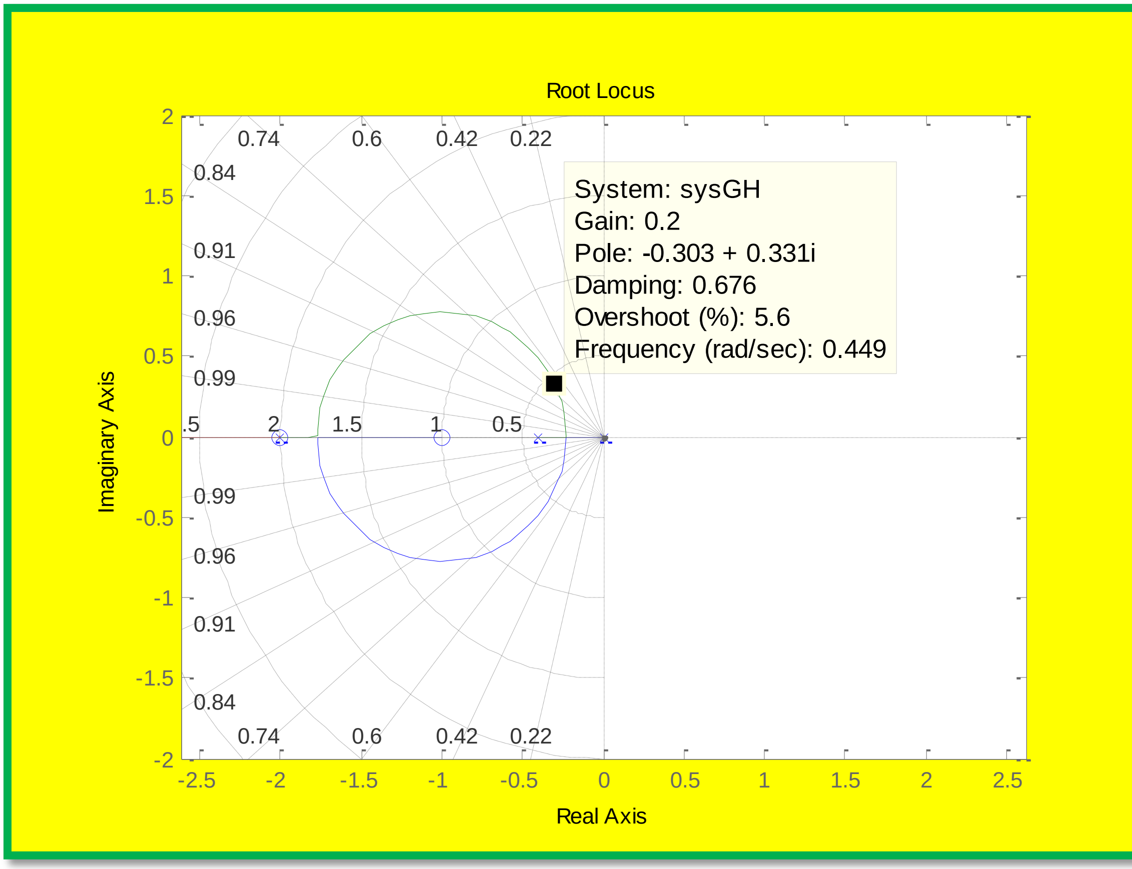
$$c_1 > 0 \to K_C\beta > 0 \to K_C > 0$$

(e) Evans root –locus Method:

Parameter	Lag Time	Lead Time	Lag Factor	Lead Factor	Moment of Inertia
Symbol	T_1	T_2	γ	β	J
Unit	[s]	[s]	[-]	[-]	$[\text{kg-m}^2]$
Data	1.0	0.5	2.0	5.0	0.2

$$K_C > 0 \rightarrow K_C = 0.2$$

$$\tau = \frac{1}{\omega_n * \zeta}\ (sec)$$



Using the MATLAB program as we obtain the Evan's root loci of the system, proportional gain for stability required is any value greater than zero as plot is on the left hand side only.

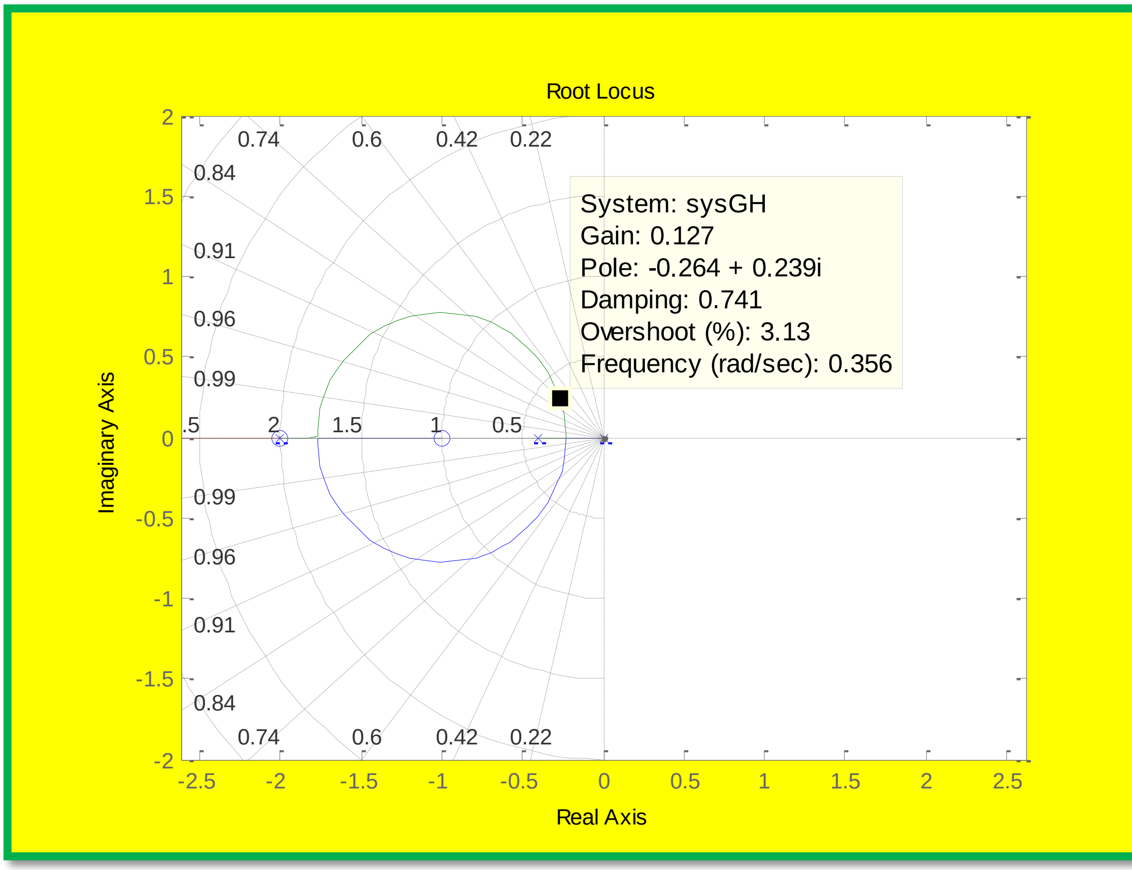
Parameters	Value
Gain (K_c)	0.2
Pole	-0.303+0.331i
Damping Ration (ζ)	0.676
Maximum Overshoot (M_p)	5.6 (%)
Damped Natural Frequency (ω_d)	0.449 (rad/sec)
Un damped Natural Frequency (ω_n)	0.826(rad/sec)

Time Constant (τ)	1.78 sec

(f) Controller design:

Case	Specification	Case	Specification
f1	damping ratio $\zeta = 0.74$	f2	undamped natural frequency $\omega_n = 1.0\ \mathrm{rad}/\mathrm{s}$
f3	damped natural frequency $\omega_d = 0.5\ \mathrm{rad}/\mathrm{s}$	f4	time constant $\tau = 2$ s
f5	maximum overshoot $M_p = 7.5\%$		

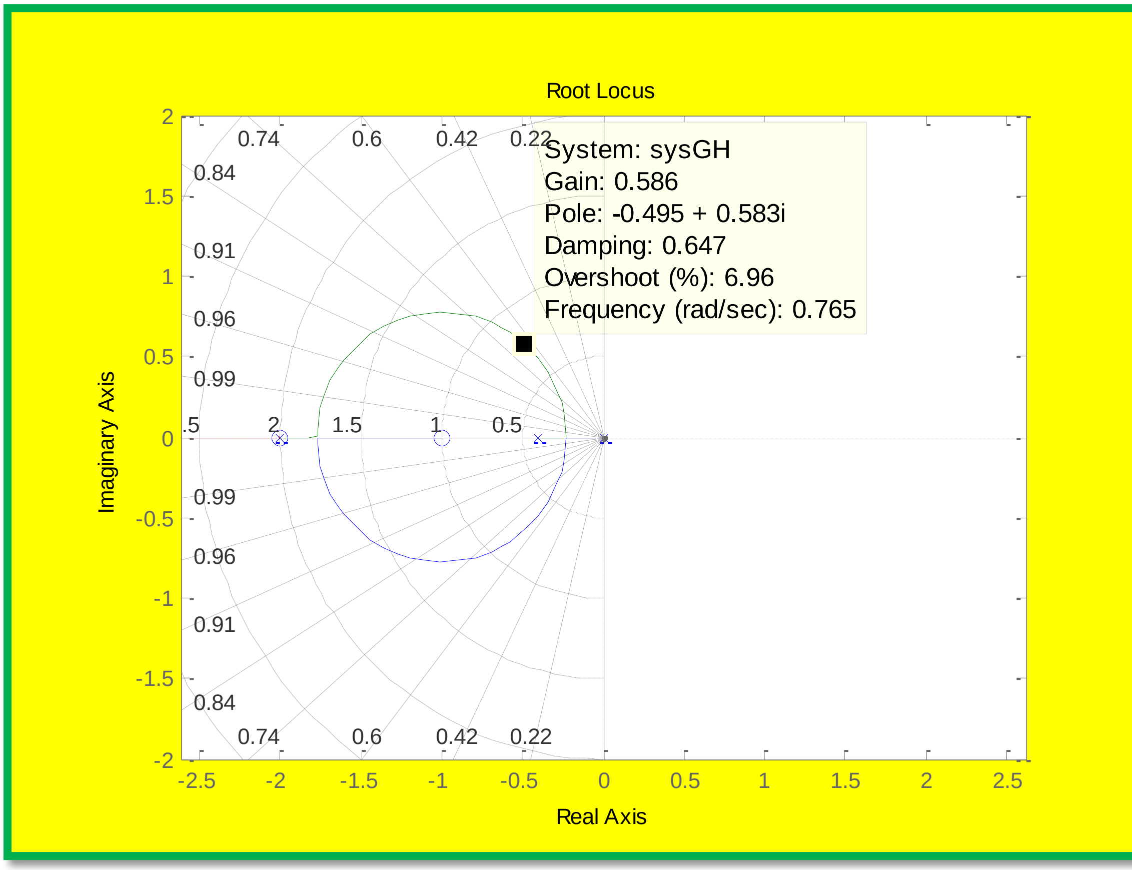
$\boldsymbol{Case\ F1}$: $(\boldsymbol{Damping\ ratio}\ \boldsymbol{\zeta = 0.74})$



Parameters	Value
Gain (K_c)	0.127
Pole	-0.264+0.2391i
Damping Ration (ζ)	0.741
Maximum Overshoot (M_p)	3.13 (%)
Damped Natural Frequency (ω_d)	0.356

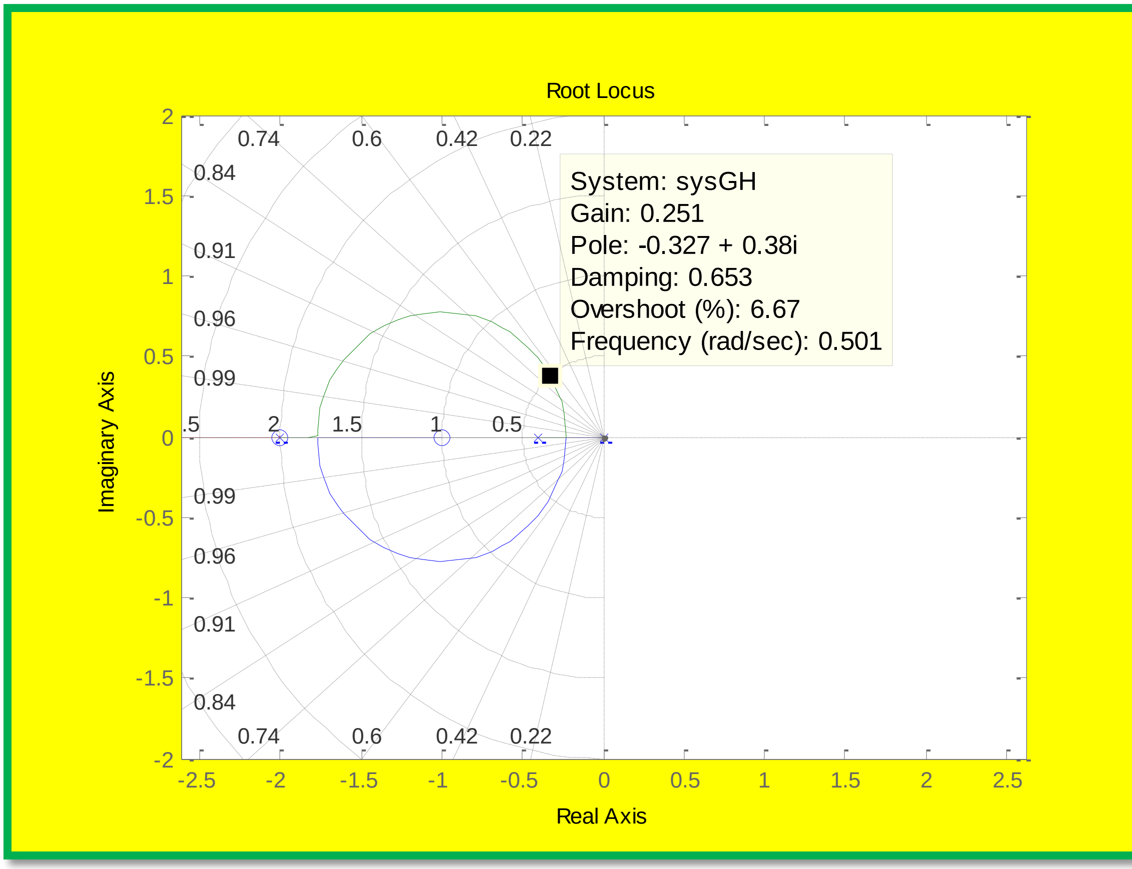
	(rad/sec)
Un damped Natural Frequency (ω_n)	0.789(rad/sec)
Time Constant (τ)	1.71 sec

Case F2: (***Undamped Natural Frequency*** $(\omega_n) = \mathbf{1.0}\ rad/s$)



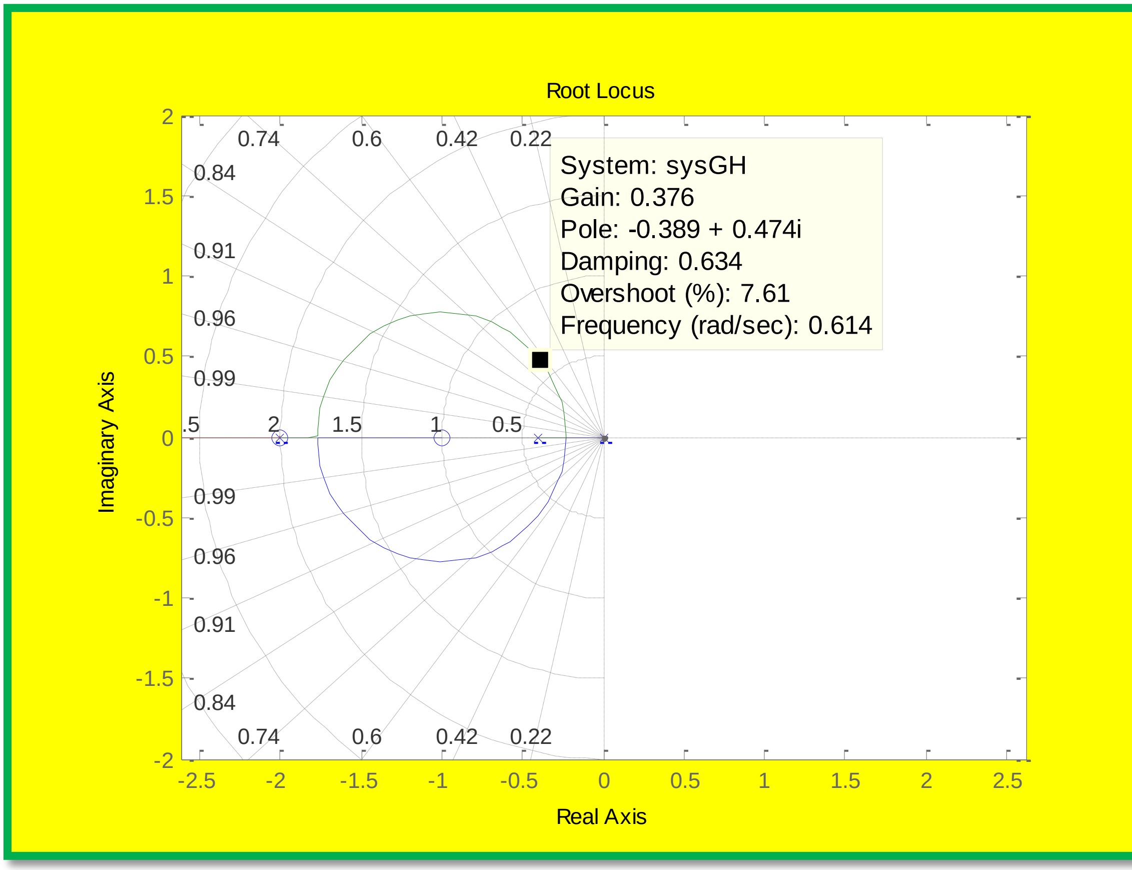
Parameters	Value
Gain (K_c)	0.586
Pole	-0.495+.583i
Damping Ration (ζ)	0.647
Maximum Overshoot (M_p)	6.96 (%)
Damped Natural Frequency (ω_d)	0.765 (rad/sec)
Un damped Natural Frequency (ω_n)	1(rad/sec)
Time Constant (τ)	1.54 (sec)

$\boldsymbol{Case\ F3}$: $(\boldsymbol{Damped\ Natural\ Frequency}\ (\omega_d) = \mathbf{0.5}\ \boldsymbol{rad/s})$



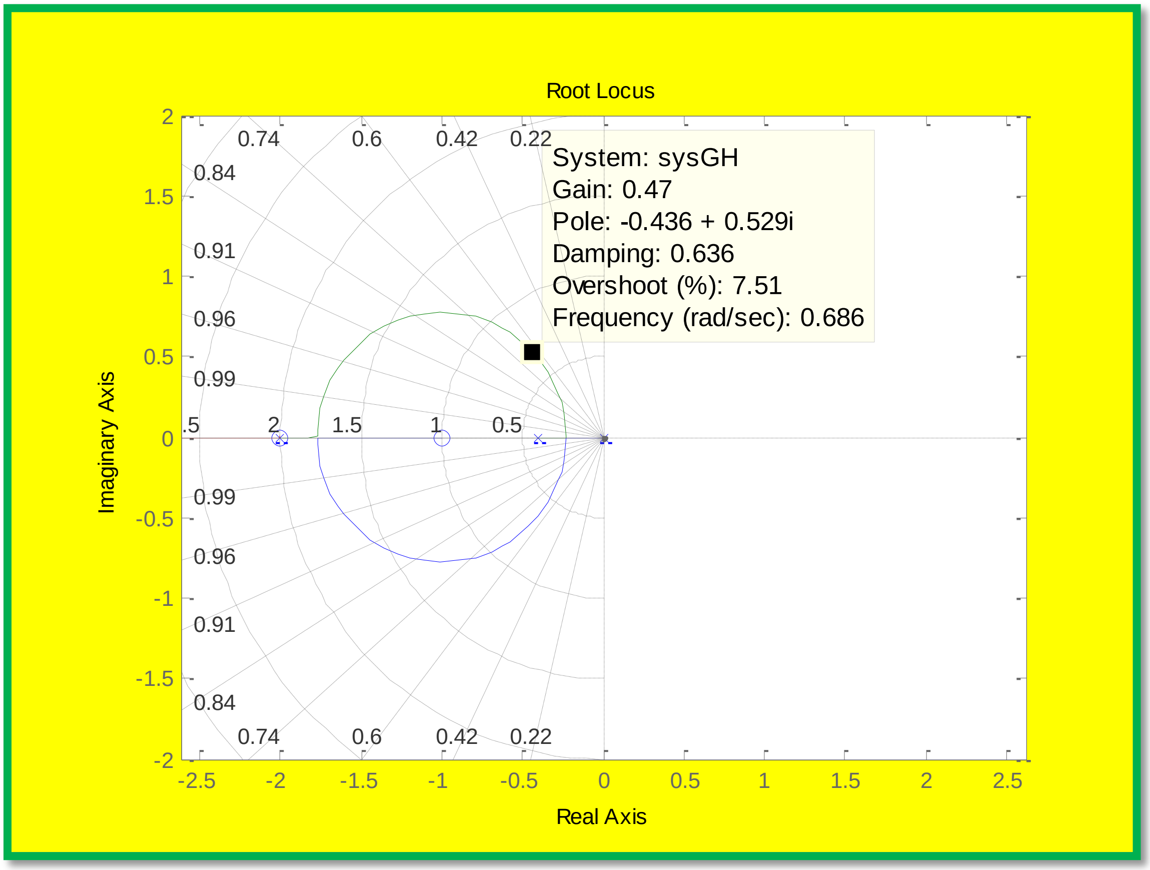
Parameters	Value
$\boldsymbol{Gain}\ (K_c)$	0.251
$\boldsymbol{Pole}$	-0.327+0.38i
$\boldsymbol{Damping\ Ration}\ (\zeta)$	0.653
$\boldsymbol{Maximum\ Overshoot}\ (M_p)$	6.67 (%)
$\boldsymbol{Damped\ Natural\ Frequency}\ (\omega_d)$	0.501 (rad/sec)
$\boldsymbol{Un\ damped\ Natural\ Frequency}\ (\omega_n)$	0.661(rad/sec)
$\boldsymbol{Time\ Constant}\ (\tau)$	2.31 (sec)

$$\textit{Case F4}: \left(\textit{Time constant } (\tau) = \frac{1}{\omega_n * \zeta} = 2s\right)$$



Parameters	Value
Gain (K_c)	0.376
Pole	-0.3891+0.474i
Damping Ration (ζ)	0.634
Maximum Overshoot (M_p)	7.61 (%)
Damped Natural Frequency (ω_d)	0.614 (rad/sec)
Un damped Natural Frequency (ω_n)	0.793(rad/sec)
Time Constant (τ)	1.99 (sec)

$\boldsymbol{Case\ F5}: (\boldsymbol{Maxumum\ Overshoot}\ (\boldsymbol{M_p}) = \boldsymbol{7.5}\%)$



Parameters	Value
$\boldsymbol{Gain\ \ (K_c)}$	0.47
$\boldsymbol{Pole}$	-0.436+0.529i
$\boldsymbol{Damping\ Ration\ (\zeta)}$	0.636
$\boldsymbol{Maximum\ Overshoot\ (M_p)}$	7.51 (%)
$\boldsymbol{Damped\ Natural\ Frequency\ (\omega_d)}$	0.686 (rad/sec)
$\boldsymbol{Un\ damped\ Natural\ Frequency\ (\omega_n)}$	0.888(rad/sec)
$\boldsymbol{Time\ Constant\ (\tau)}$	1.76 (sec)

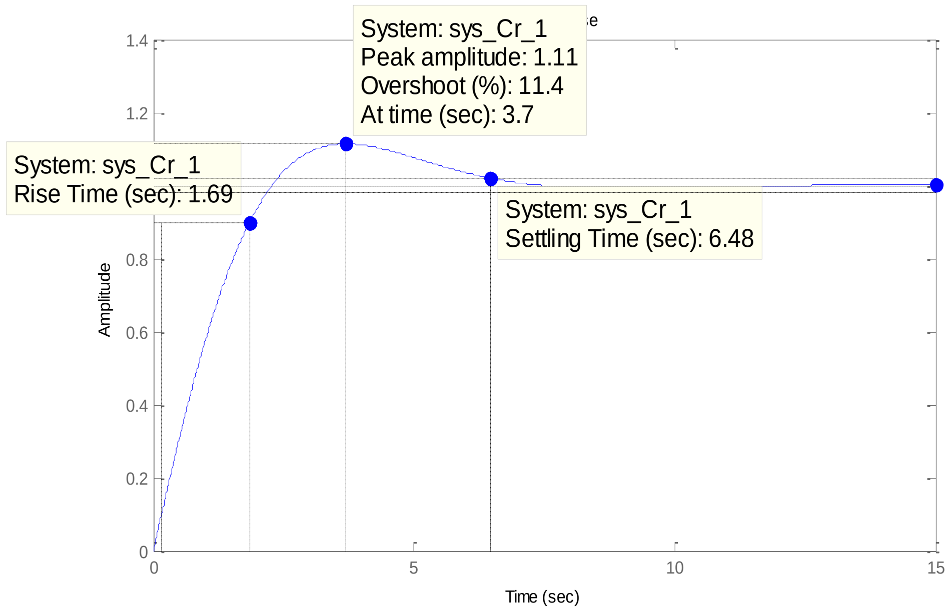
(g) Unit-step responses:

g1) From part (c) and Part (f)

Generally for unit step response steady state $(\pm 2\%)$ $is\ at\ 4\tau$.

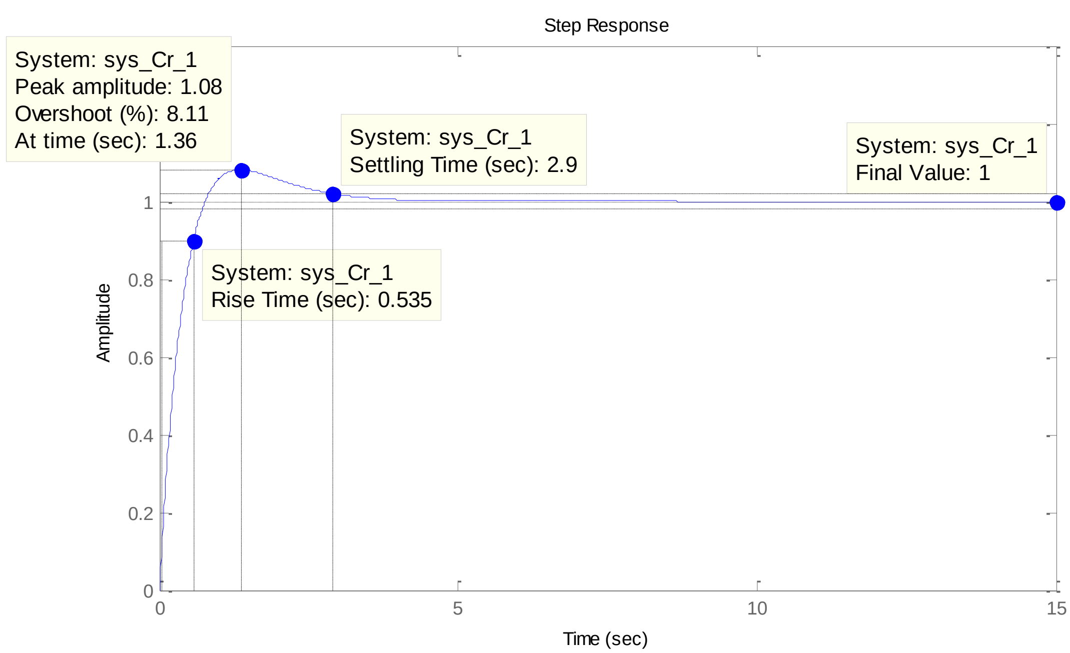
Case	1	2	3	4	5
K_c	0.127	0.586	0.251	0.376	0.47
$\frac{w_{c_{ss}}}{\overline{w_r}}$	1	1	1	1	1
$\frac{w_{e_{ss}}}{\overline{w_r}}$	0	0	0	0	0
τ (sec)	1.71	1.54	2.31	1.99	1.76
4τ (sec)	6.84	6.16	9.24	7.96	7.04
$M_p(\%)$	3.13	6.96	6.67	7.61	7.51
$\omega_d\left(\frac{rad}{s}\right)$	0.356	0.765	0.501	0.614	0.686

Case 1



ω_d	0.84
M_p	11.4%
τ	1.62

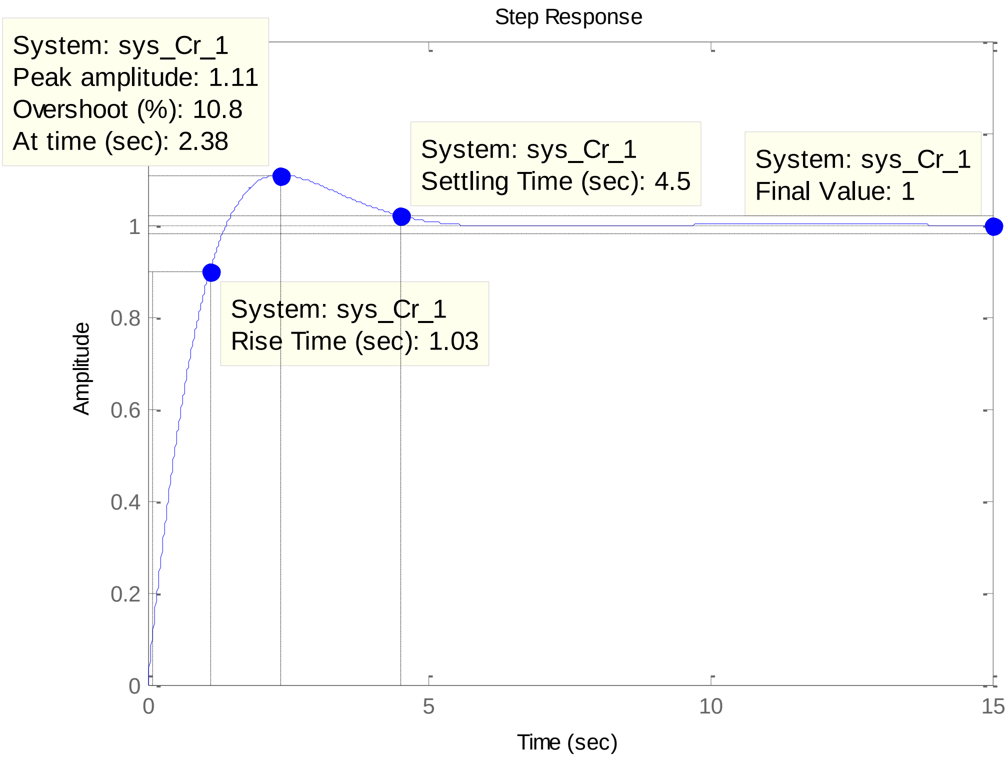
Case 2



ω_d	2.30
M_p	8%
τ	0.725

Case 3

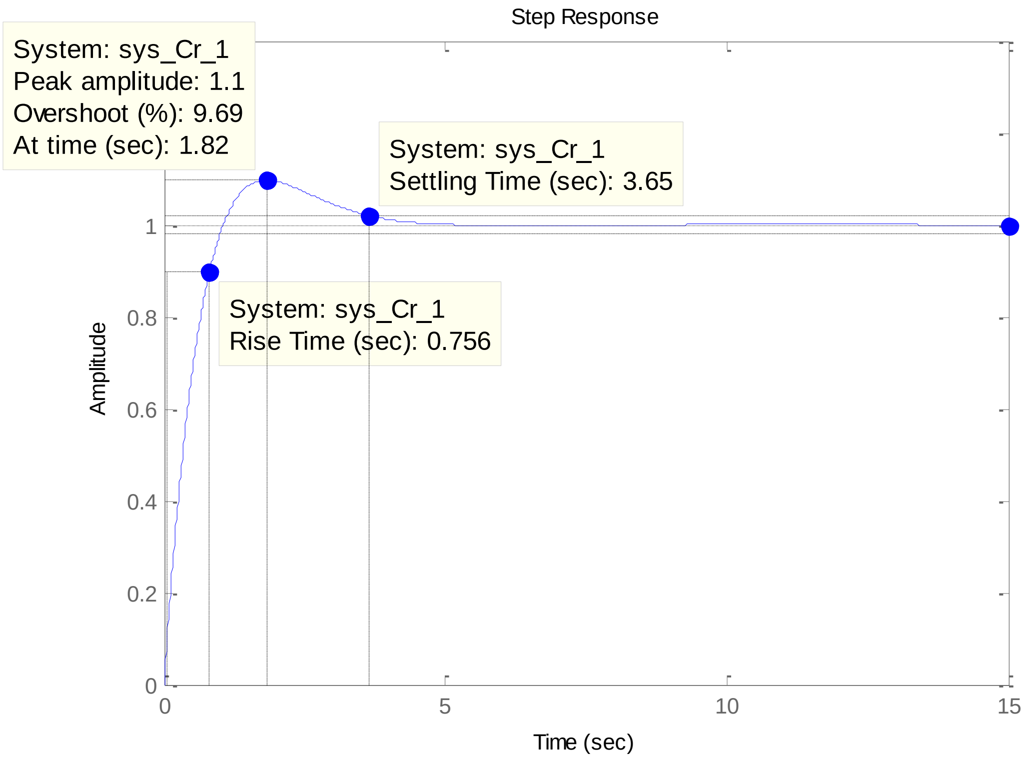
ω_d	1.31
M_p	10.8%



$\mathbf{1.12}$

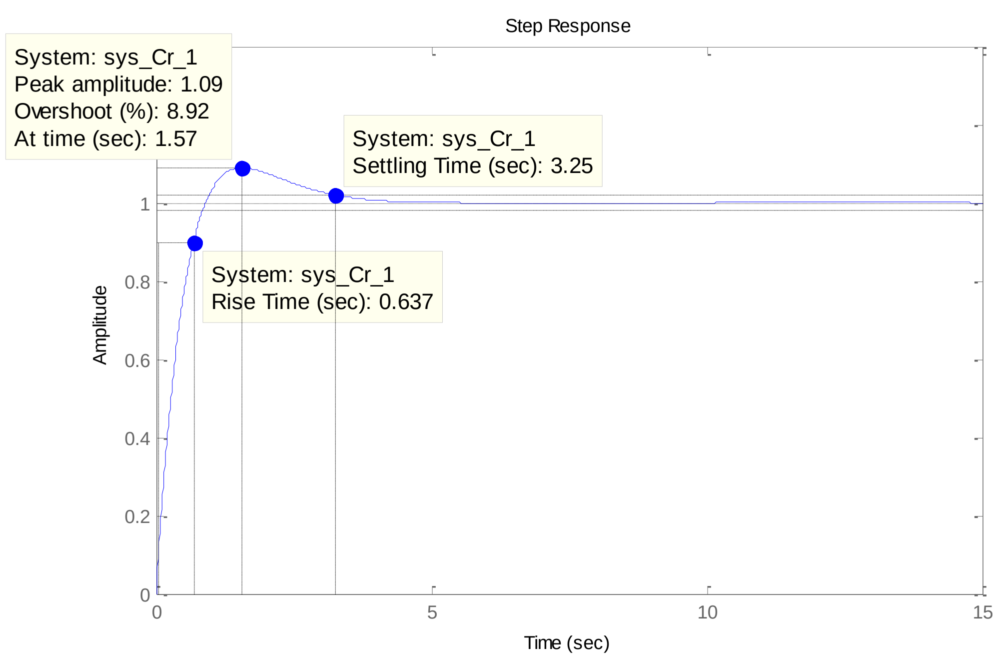
Case 4

$\boldsymbol{\omega_d}$	$\mathbf{1.72}$
$\boldsymbol{M_p}$	$\mathbf{9.7\%}$
$\boldsymbol{\tau}$	

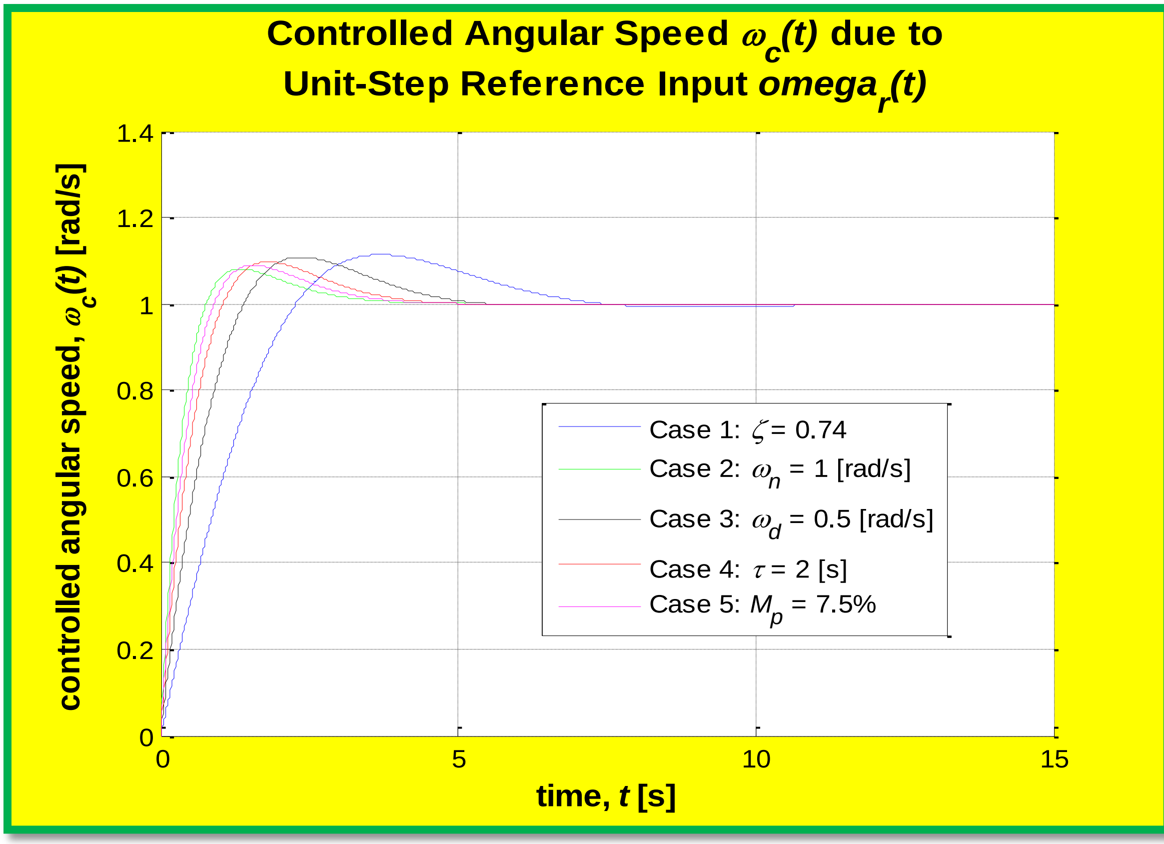


$\mathbf{0.912}$

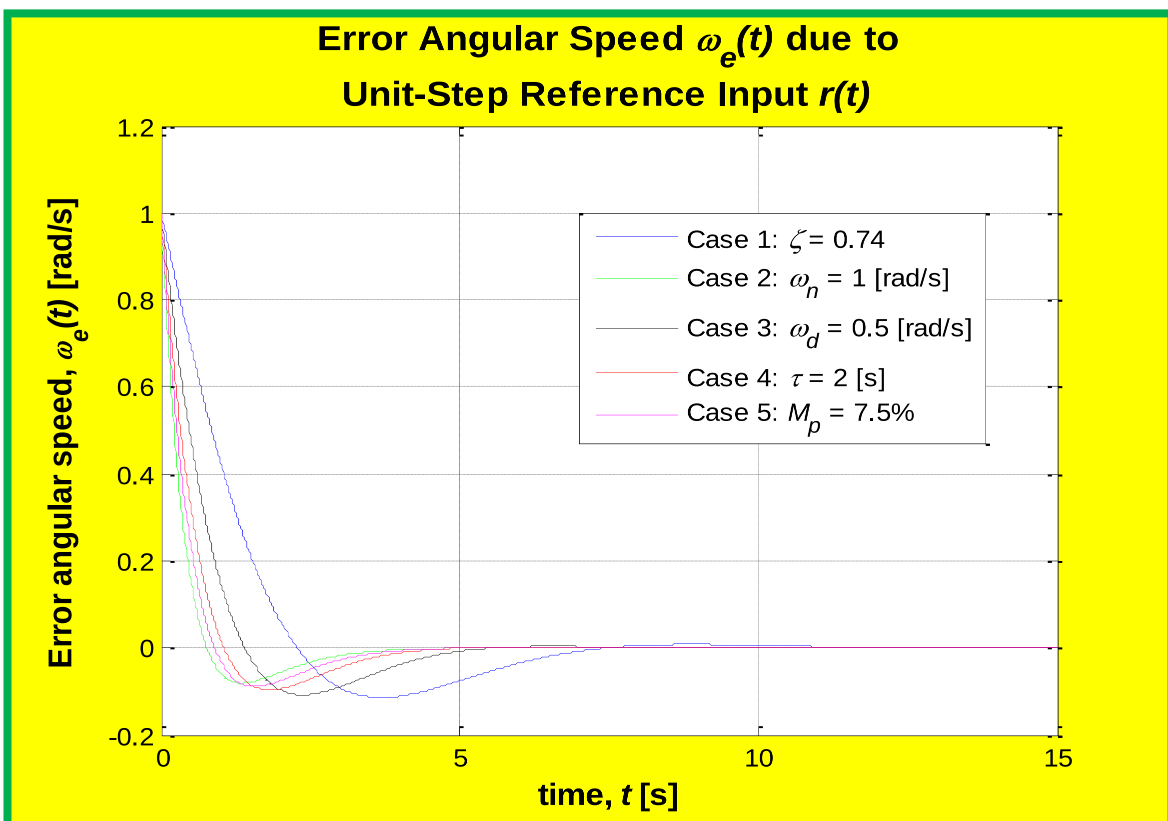
Case5



ω_d	$\mathbf{2.0}$
M_p	$\mathbf{8.9\%}$
τ	$\mathbf{0.812}$



Controlled Angular Speed ω_c(t) due to
Unit-Step Reference Input omega_r(t)
controlled angular speed, ω_c(t) [rad/s]
time, t [s]
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
Case 1: ζ = 0.74
Case 2: ω_n = 1 [rad/s]
Case 3: ω_d = 0.5 [rad/s]
Case 4: τ = 2 [s]
Case 5: M_p = 7.5%

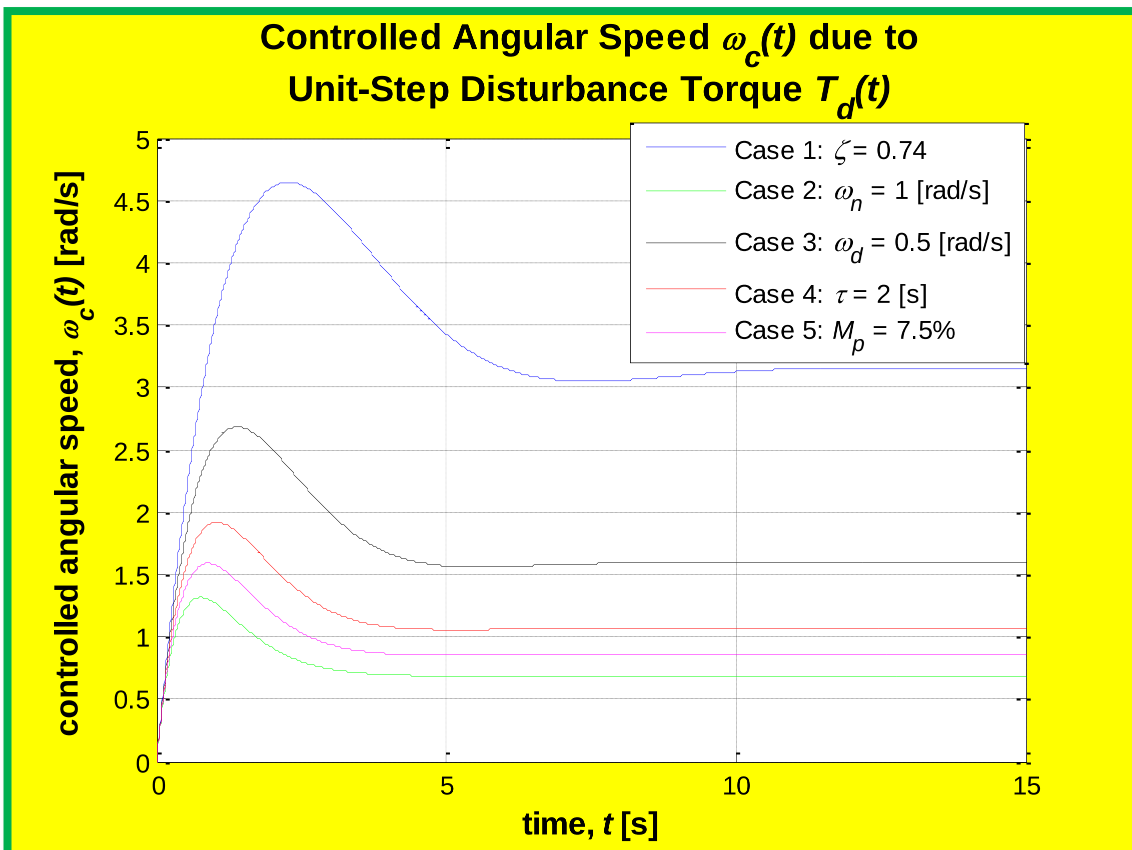


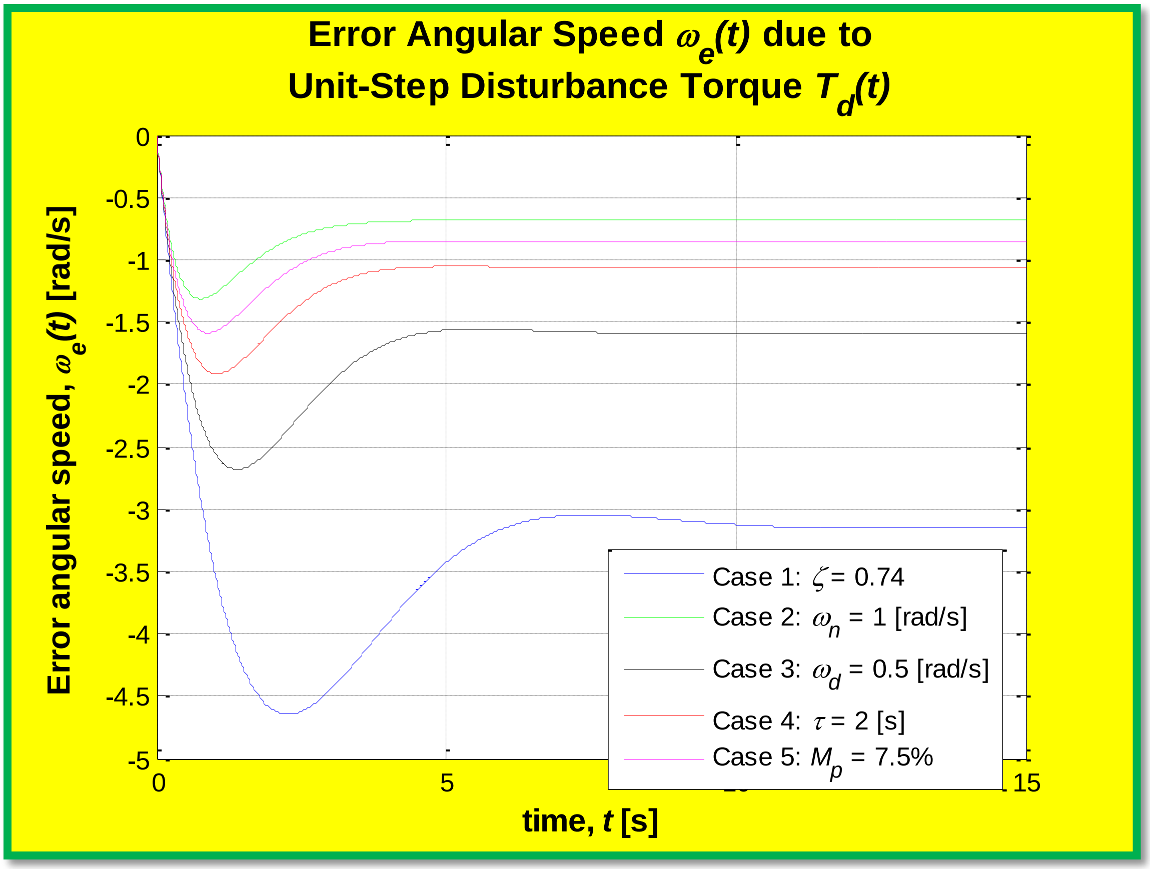
Error Angular Speed ω_e(t) due to
Unit-Step Reference Input r(t)
Error angular speed, ω_e(t) [rad/s]
time, t [s]
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
5
10
15
Case 1: ζ = 0.74
Case 2: ω_n = 1 [rad/s]
Case 3: ω_d = 0.5 [rad/s]
Case 4: τ = 2 [s]
Case 5: M_p = 7.5%

In the figures for unit reference input for all the cases the has been achieved the goal of "regulation/tracking". The steady-state controlled speed : $\omega_{c_{ss}}(t \to \infty) = \overline{\omega_r}$, which is consistent with the prediction from Part (c). For each case, it takes approximately (5 s) to reach the steady state value and also error angular speed goes to zero.

g2) From part (c)

Case	1	2	3	4	5
K_c	0.127	0.586	0.251	0.376	0.47
$\frac{w_{c_{ss}}}{T_d}$	3.14	0.68	1.59	1.06	0.85
$\frac{w_{e_{ss}}}{T_d}$	-3.14	-0.68	-1.59	-1.06	-0.85

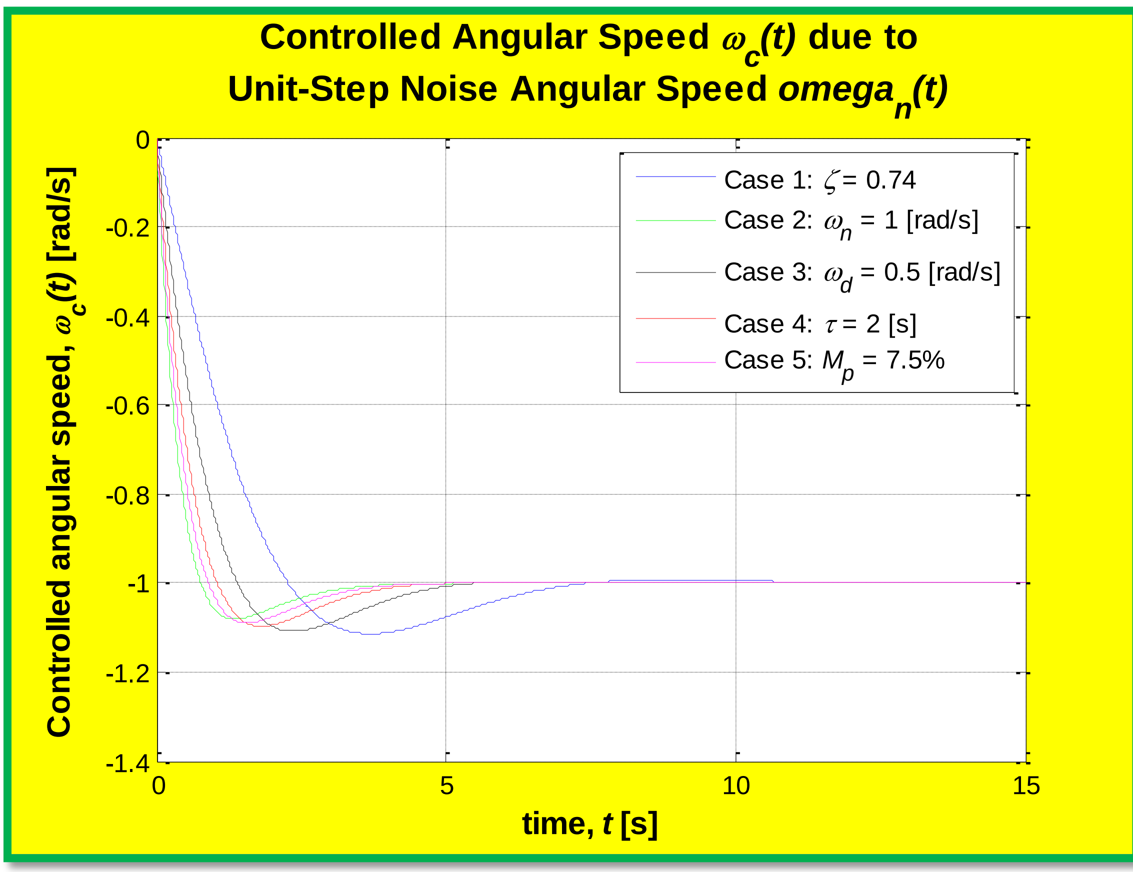


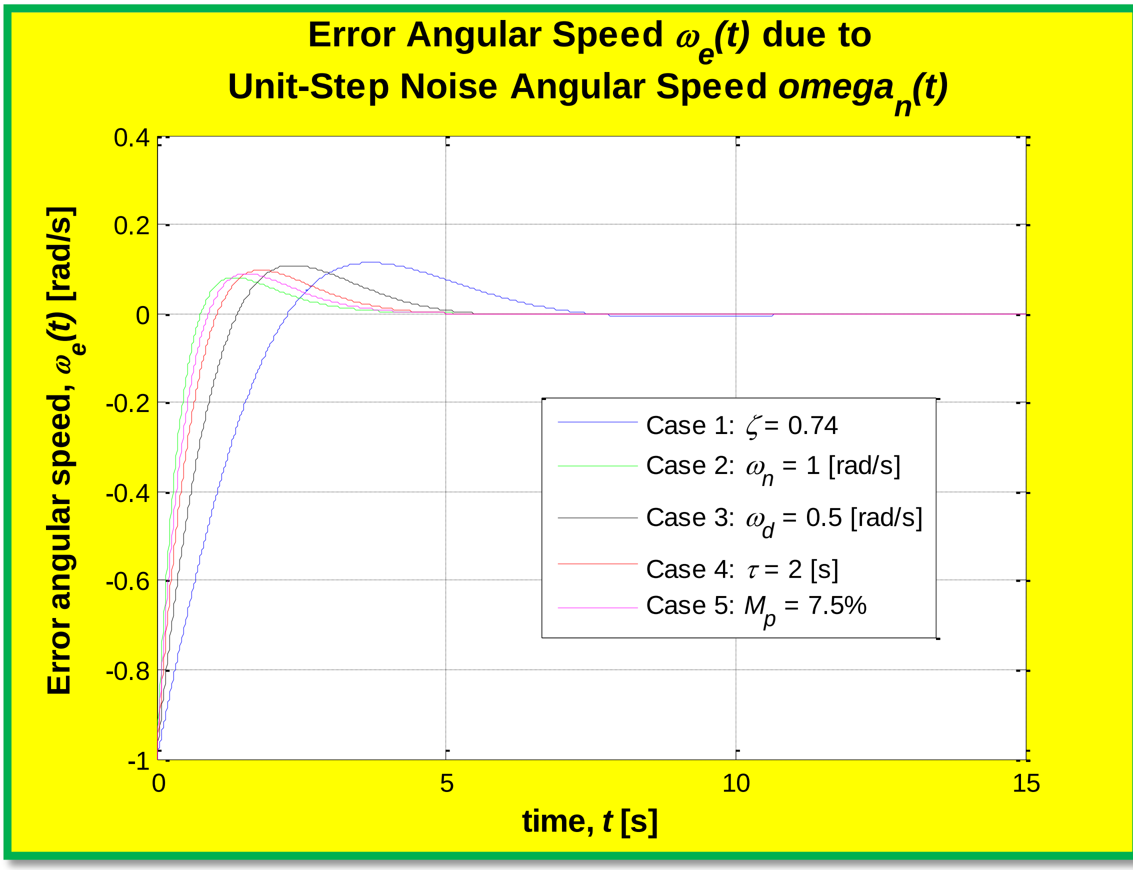


In the figures for unit Disturbance torque for all the cases has not been achieved of goal of "disturbance rejection". The steady-state controlled speed is: $\omega_{c_{ss}}(t \to \infty) \neq 0$, which is consistent with the prediction from Part (c). For each case, it takes approximately (5 s) to reach the steady state value none zero value hence it not a very good disturbance rejection system.

G3) From part (c)

Case	1	2	3	4	5
K_c	0.127	0.586	0.251	0.376	0.47
$\frac{w_{c_{ss}}}{w_n}$	-1	-1	-1	-1	-1
$\frac{w_{e_{ss}}}{w_n}$	0	0	0	0	0





As we can see in the figures, again none of the cases can achieve the goal of "noise rejection". The remnant controlled speed $\omega_{c_{ss}}(t \to \infty) = -\overline{\omega_n}$, which is consistent with the prediction from Part (c). For each case, it takes approximately (6 s) to reach the steady state value none zero value hence it not a very good noise rejection system.

(h) Appendix:

Evans root-locus method

```
clc
clear all
%%Lag T_1 and Lead times T_2  [seconds]
T_1=1.0;
T_2=0.5;
%%Lag Factor
gamma=2.0;
%%Lead Factor
beta=5.0;
%%Moment of Inertia [kg-m^2]
J=0.2;
%% Control Gain
K_C=.2;
% Evans root loci
numGc=[K_C*beta*T_2*T_1,K_C*beta*(T_2+T_1),K_C*beta];
denGc=[T_2*T_1*beta,T_2*beta*gamma+T_1,gamma];
sysGc=tf(numGc,denGc);
numGp=1;
denGp=[J,0];
sysGp=tf(numGp,denGp);
numG=conv(numGc,numGp);
denG=conv(denGc,denGp);
root_z=roots(numG);
root_p=roots(denG);
sysG=series(sysGc,sysGp);
numH=1;
denH=1;
sysH=tf(numH,denH);
sysGH=series(sysG,sysH);
rlocus(sysGH)
axis([-2,2,-2,2]);axis('equal');sgrid
```

Unit Step Response

```
clc
clear all

% Define Parameter
T_1=1;T_2=0.5;
gamma=2.0;beta=5.0;
J=0.2;
n_case=5;n_step=1501;
t_min=0;t_max=15;
delta_t=(t_max-t_min)/(n_step-1);
```

```
t=[t_min:delta_t:t_max];

% Define Lead Lag Compensator gain
K_C(1,1)=0.127;K_C(2,1)=0.586;K_C(3,1)=0.251;K_C(4,1)=0.376;K_C(5,1)=0.47;




for c=1:n_case
    num_Cr=[K_C(c,1)*beta*T_1*T_2, (T_1+T_2)*K_C(c,1)*beta, K_C(c,1)*beta];

den_Cr=[J*beta*T_1*T_2,(T_1*J+gamma*J*beta*T_2+K_C(c,1)*beta*T_1*T_2),(J*gamma+K_C(c,1
)*beta*T_1+K_C(c,1)*beta*T_2),K_C(c,1)*beta];
    sys_Cr=tf(num_Cr,den_Cr);
    step_respCr(:,c)=step(sys_Cr,t);
end
hold on
grid on
figure (1)
plot(t,step_respCr(:,1),'b-',...
     t,step_respCr(:,2),'g-',...
     t,step_respCr(:,3),'k-',...
     t,step_respCr(:,4),'r-',...
     t,step_respCr(:,5),'m-');
legend('Case 1: \it\zeta \rm= 0.74','Case 2: \it\omega_n \rm= 1 [rad/s]','Case 3:
\it\omega_d \rm= 0.5 [rad/s]','Case 4: \it\tau \rm= 2 [s]','Case 5: \itM_p \rm= 7.5%')
title({'\fontsize{14}\bfControlled Angular Speed \it{\omega_c(t)} \rm\bfdue to';...
    '\fontsize{14}\bfUnit-Step Reference Input \it{omega_r(t)}'})
xlabel('\fontsize{12}\bftime, \it{t} \rm\bf{[s]}')
ylabel('\fontsize{12}\bfcontrolled angular speed, \it{\omega_c(t)} \rm\bf{[rad/s]}')
%
for c=1:n_case
    num_Er=[J*T_1*T_2*beta,(J*T_1+gamma*beta*J*T_2),J*gamma,0];

den_Er=[J*beta*T_1*T_2,(T_1*J+gamma*J*beta*T_2+K_C(c,1)*beta*T_1*T_2),(J*gamma+K_C(c,1
)*beta*T_1+K_C(c,1)*beta*T_2),K_C(c,1)*beta];
    sys_Er=tf(num_Er,den_Er);
    step_respEr(:,c)=step(sys_Er,t);
end
hold on
grid on
figure (2)
plot(t,step_respEr(:,1),'b-',...
     t,step_respEr(:,2),'g-',...
     t,step_respEr(:,3),'k-',...
     t,step_respEr(:,4),'r-',...
     t,step_respEr(:,5),'m-');
legend('Case 1: \it\zeta \rm= 0.74','Case 2: \it\omega_n \rm= 1 [rad/s]','Case 3:
\it\omega_d \rm= 0.5 [rad/s]','Case 4: \it\tau \rm= 2 [s]','Case 5: \itM_p \rm= 7.5%')
title({'\fontsize{14}\bfError Angular Speed \it{\omega_e(t)} \rm\bfdue to';...
    '\fontsize{14}\bfUnit-Step Reference Input \it{r(t)}'})
xlabel('\fontsize{12}\bftime, \it{t} \rm\bf{[s]}')
ylabel('\fontsize{12}\bfError angular speed, \it{\omega_e(t)} \rm\bf{[rad/s]}')
%
for c=1:n_case
    num_Cd=[T_1*T_2*beta, (T_1+gamma*beta*T_2),gamma];
```

```
den_Cd=[J*beta*T_1*T_2,(T_1*J+gamma*J*beta*T_2+K_C(c,1)*beta*T_1*T_2),(J*gamma+K_C(c,1
)*beta*T_1+K_C(c,1)*beta*T_2),K_C(c,1)*beta];
    sys_Cd=tf(num_Cd,den_Cd);
    step_respCd(:,c)=step(sys_Cd,t);
end
hold on
grid on
figure (3)
plot(t,step_respCd(:,1),'b-',...
     t,step_respCd(:,2),'g-',...
     t,step_respCd(:,3),'k-',...
     t,step_respCd(:,4),'r-',...
     t,step_respCd(:,5),'m-');
legend('Case 1: \it\zeta \rm= 0.74','Case 2: \it\omega_n \rm= 1 [rad/s]','Case 3:
\it\omega_d \rm= 0.5 [rad/s]','Case 4: \it\tau \rm= 2 [s]','Case 5: \itM_p \rm= 7.5%')
title({'\fontsize{14}\bfControlled Angular Speed \it{\omega_c(t)} \rm\bfdue to';...
    '\fontsize{14}\bfUnit-Step Disturbance Torque \it{T_d(t)}'})
xlabel('\fontsize{12}\bftime, \it{t} \rm\bf{[s]}')
ylabel('\fontsize{12}\bfcontrolled angular speed, \it{\omega_c(t)} \rm\bf{[rad/s]}')
%
for c=1:n_case
    num_Ed=[-T_1*T_2*beta,-(T_1+gamma*beta*T_2),-gamma];

den_Ed=[J*beta*T_1*T_2,(T_1*J+gamma*J*beta*T_2+K_C(c,1)*beta*T_1*T_2),(J*gamma+K_C(c,1
)*beta*T_1+K_C(c,1)*beta*T_2),K_C(c,1)*beta];
    sys_Ed=tf(num_Ed,den_Ed);
    step_respEd(:,c)=step(sys_Ed,t);
end
hold on
grid on
figure (4)
plot(t,step_respEd(:,1),'b-',...
     t,step_respEd(:,2),'g-',...
     t,step_respEd(:,3),'k-',...
     t,step_respEd(:,4),'r-',...
     t,step_respEd(:,5),'m-');
legend('Case 1: \it\zeta \rm= 0.74','Case 2: \it\omega_n \rm= 1 [rad/s]','Case 3:
\it\omega_d \rm= 0.5 [rad/s]','Case 4: \it\tau \rm= 2 [s]','Case 5: \itM_p \rm= 7.5%')
title({'\fontsize{14}\bfError Angular Speed \it{\omega_e(t)} \rm\bfdue to';...
    '\fontsize{14}\bfUnit-Step Disturbance Torque \it{T_d(t)}'})
xlabel('\fontsize{12}\bftime, \it{t} \rm\bf{[s]}')
ylabel('\fontsize{12}\bfError angular speed, \it{\omega_e(t)} \rm\bf{[rad/s]}')
%
for c=1:n_case
    num_Cn=[-K_C(c,1)*beta*T_1*T_2,-(T_1+T_2)*K_C(c,1)*beta,-K_C(c,1)*beta(0];

den_Cn=[J*beta*T_1*T_2,(T_1*J+gamma*J*beta*T_2+K_C(c,1)*beta*T_1*T_2),(J*gamma+K_C(c,1
)*beta*T_1+K_C(c,1)*beta*T_2),K_C(c,1)*beta];
    sys_Cn=tf(num_Cn,den_Cn);
    step_respCn(:,c)=step(sys_Cn,t);
end
hold on
grid on
figure (5)
plot(t,step_respCn(:,1),'b-',...
     t,step_respCn(:,2),'g-',...
     t,step_respCn(:,3),'k-',...
     t,step_respCn(:,4),'r-',...
     t,step_respCn(:,5),'m-');
```

```
legend('Case 1: \it\zeta \rm= 0.74','Case 2: \it\omega_n \rm= 1 [rad/s]','Case 3:
\it\omega_d \rm= 0.5 [rad/s]','Case 4: \it\tau \rm= 2 [s]','Case 5: \itM_p \rm= 7.5%')
title({'\fontsize{14}\bfControlled Angular Speed \it{\omega_c(t)} \rm\bfdue to';...
    '\fontsize{14}\bfUnit-Step Noise Angular Speed \it{omega_n(t)}'})
xlabel('\fontsize{12}\bftime, \it{t} \rm\bf{[s]}')
ylabel('\fontsize{12}\bfControlled angular speed, \it{\omega_c(t)} \rm\bf{[rad/s]}')
%
for c=1:n_case
    num_En=[-J*T_1*T_2*beta,-(J*T_1+gamma*beta*J*T_2),-J*gamma,0];

den_En=[J*beta*T_1*T_2,(T_1*J+gamma*J*beta*T_2+K_C(c,1)*beta*T_1*T_2),(J*gamma+K_C(c,1
)*beta*T_1+K_C(c,1)*beta*T_2),K_C(c,1)*beta];
    sys_En=tf(num_En,den_En);
    step_respEn(:,c)=step(sys_En,t);
end
hold on
grid on
figure (6)
plot(t,step_respEn(:,1),'b-',...
     t,step_respEn(:,2),'g-',...
     t,step_respEn(:,3),'k-',...
     t,step_respEn(:,4),'r-',...
     t,step_respEn(:,5),'m-');
legend('Case 1: \it\zeta \rm= 0.74','Case 2: \it\omega_n \rm= 1 [rad/s]','Case 3:
\it\omega_d \rm= 0.5 [rad/s]','Case 4: \it\tau \rm= 2 [s]','Case 5: \itM_p \rm= 7.5%')
title({'\fontsize{14}\bfError Angular Speed \it{\omega_e(t)} \rm\bfdue to';...
    '\fontsize{14}\bfUnit-Step Noise Angular Speed \it{omega_n(t)}'})
xlabel('\fontsize{12}\bftime, \it{t} \rm\bf{[s]}')
ylabel('\fontsize{12}\bfError angular speed, \it{\omega_e(t)} \rm\bf{[rad/s]}')
grid ; hold off
```